

Physics 123

Lab Report Quick Reference

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1 Quantify

Science is quantitative. Every important scientific result or comparison is *numerical*.

This is much different than the arts (painting, music, literature, etc.), which are intrinsically *qualitative* (non-numerical). Many people may agree that painting #1 is better than painting #2, agreeing even on the specific qualities that make #1 superior, but it would be difficult for them to express the “goodness” of each painting with a *universally* accepted number (not a rating, but some other quantitative property like “creativity factor”). In fact, there’s probably a subset of people who actually prefer painting #2. This makes art largely *subjective* (in the eye of the beholder).

But science attempts to find *universal* truths, and a truth cannot be subject to interpretation. This requires scientists to be *objective* — making judgments based upon facts, and aggressively removing all sources of bias (in either the facts or the judgments). When scientists publish their results, they are claiming to report a truth. If they are wrong (even unintentionally), their “alternative truths” can have dire consequences (e.g. the unethical research studies funded by tobacco companies which claimed that cigarettes were neither addictive nor dangerous, convincing millions not to quit smoking).

Quantitative analysis and objective truth are intimately related because numbers can be compared to one another in an objective way. It doesn’t matter *where* you learned math, $5 > 3$ and $3.01 \approx 3$. Thus, an argument is always made stronger when it is clad with numbers, because the concepts of “smaller”, “closer” and “better” can be given a clear definition.

1.1 Quantify, quantify, quantify!

1. Since your experimental results are science, and science is quantitative, **your experimental results must also be quantitative**. If you are attempting to demonstrate the validity of a model, then you need to do so with numbers. Often this will involve verifying that the slope of a fit line matches theoretical predictions and the model fits the data well ($R^2 \approx 1$).

Imagine we are letting a 450 g cart accelerate down an inclined track, in order to verify Newton’s second law

$$F = m a . \tag{1}$$

If we plot the cart’s *theoretically calculated* gravitational force against its *experimentally measured* acceleration (F vs. a , see Fig. 1), we can fit Newton’s second law to the data.

This will allow us to discuss how well the model fits the data. In doing so, we must support every qualitative statement with numerical/quantitative evidence.

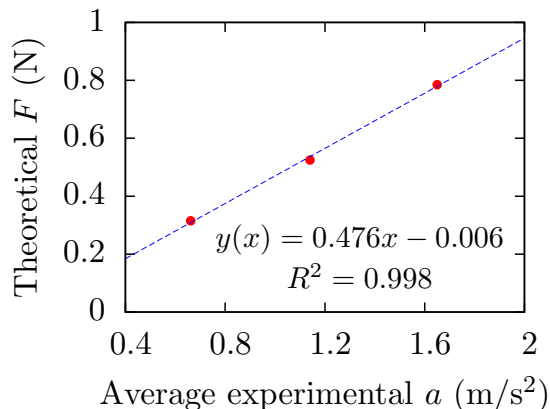


Figure 1: F vs. a for a 450 g cart

- (a) A **purely qualitative** analysis (like the following example) is unacceptable:

“It is clear from Fig. 1 that Eq. 1 does a good job of modeling the data.”

A graph showing a “clear” qualitative trend will certainly possess an $R^2 \approx 1$, otherwise the trend would not be “clear” at all. A proper statement is:

“It is clear from Fig. 1 that Eq. 1 does a good job of modeling the data, because the R^2 of the fit line (0.998) is very close to perfect ($R^2 = 1$).”

Here, the word “close” refers to the fact that 0.998 and 1 differ by 0.2% (see Sec. 1.4.2 about determining a good R^2).

- (b) Your “results” are numbers. Discussing your results requires **including the numbers in the discussion**. Thus, an even better discussion paragraph would say:

“It is clear from Fig. 1 that Eq. 1 does a good job of modeling the data, because the R^2 of the fit line (0.998) is very close to perfect ($R^2 = 1$). Additionally, the slope of the fit line indicates that the mass of our cart is 0.476 kg, which is quite close to the mass we measured with the digital balance, 0.450 kg.”

However, this could still use some improvement, since “quite close” is a bit wobbly.

2. A great way to quantify agreement between an experimental measurement ($x_{\text{experiment}}$) and some reference value ($x_{\text{reference}}$) is via their relative difference

$$\text{RelDiff} = \frac{x_{\text{experiment}} - x_{\text{reference}}}{x_{\text{reference}}} . \quad (2)$$

Here, x could be any observable, not just distance. This allows us to create an optimal discussion paragraph:

“It is clear from Fig. 1 that Eq. 1 does a good job of modeling the data, because the R^2 of the fit line (0.998) is very close to perfect ($R^2 = 1$). Additionally, the slope of the fit line indicates that the mass of our cart is 0.476 kg, which is quite close (+5.8%) to the mass we measured with the digital balance, 0.450 kg.”

A +6% result is pretty good; the model ($F = ma$) fits the data well, with multiple quantitative supports, especially since the model we used completely ignores friction.

- (a) Notice that the **sign** of the relative difference is important, since it tells you whether the experimental measurement was low or high. The sign can help indicate a systematic cause for the discrepancy (e.g. friction always slows things down, so its effects have a systematic direction).
3. If the lab manual ever asks you to **show** something, then you need to **show quantitatively**, using your data from the lab. If you don't know how to do this, then you should come see me during my office hours (or send me an email).

1.2 Multiple trials, multiple configurations

Good science is repeatable because truths are universal. This means that if someone on the other side of the world repeats your experiment, they should get the same results. In physics, we are interested in studying the physical behavior of systems (springs exerting force, carts accelerating down inclines, etc.). In order to fully understand such systems, we have to place them in multiple configurations. Here, a “configuration” is a definite, repeatable state: a spring stretched to 15 cm, or a cart weighing 450 g raised to the top of a 10 degree incline and released from rest. Only by studying a large number of configurations can we begin to understand the system as a whole.

A “trial” is one test of a given configuration, with some data recorded. One trial per configuration is not sufficient. The system must be reset to the *same* configuration and the *same* measurements must be repeated. These extra trials help nullify **random errors** that crop up in both the system under study and the measuring devices used. Since random errors cause each trial to be slightly different, averaging over multiple trials tends to wash away the random errors, giving you a better picture of the true nature of each configuration.

1. In this course, you are expected to study **at least 3 different configurations per independent variable**. This is because we will be using fit lines to extract our results, and each configuration represents a point on the graph. If there were only 2 configurations, there would only be 2 points, and the fit line would be perfect (because it takes two points to characterize a line, so there will always be a perfect fit).
2. In order measure each configuration well, you are expected to take **at least 3 trials per configuration**. If the *coefficient of variation* c_v of the trials is not small ($c_v < 0.05$), the system is not behaving repeatably, and your data is infected with random errors. This will make it very hard to get a good result.
 - (a) The coefficient of variation c_v characterizes the magnitude of random errors for each set of trials. A “small” c_v is less than 5%, and is calculated like so:

- i. Given a set of N measurements $X = \{x_1, x_2, \dots, x_N\}$, you should be familiar with their *mean* \bar{x}

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i, \quad (3)$$

which is easily computed with the Excel function `AVERAGE()`.

- ii. The *standard deviation* adds up each measurement's deviation from the mean, to quantify the variability of the sample

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2}, \quad (4)$$

and is easily computed with the Excel function `STDEV.P()`.

- iii. The farther each number is from the mean, the larger the standard deviation; a set of data with lots of variation will have a large σ . However, "large" depends on the absolute size of \bar{x} . Thus, σ should be compared to the mean, so that $\sigma = 10$ from $\bar{x} = 10^3$ is revealed to indicate *small* random variations. Scaling σ by \bar{x} gives us the *coefficient of variation*

$$c_v = \frac{\sigma}{\bar{x}}. \quad (5)$$

A c_v less than 5% indicates small random errors.

1.3 Significant figures (sig. figs.)

Significant figures keep scientific results meaningful. If I have 2 carts, each with 5 kg of dirt, then I have 10 kg of dirt. Writing this as 10.0000000 kg implies that I measured the dirt with great accuracy, picking out the excess with tweezers. But I didn't do that; the extra digits are misleading. Similarly, if I weigh myself three times during the week and wind up with 250 lb, 245 lb, 248 lb, then when I take the average (246.6666666... lb), I should not report all those repeating 6s, since the bathroom scale I used is only accurate to three sig. figs. The correct average is 247 lb.

1. When you report your results in your lab report, you need to use the correct number of sig. figs. Otherwise, your results are misleading.
2. The number of sig. figs. depends on the precision of the measuring device. If a scale says 1.045 g, then there are 4 sig. figs. Since the number of sig. figs. in your results depend on the sig. figs. of the raw data from the lab, make sure you're recording as many sig. figs. as you can. For example, if your calipers have a Vernier scale, you should measure and record all 3 digits after the decimal. Similarly, if you are taking data from the computer, make certain that the **last two** digits displayed are changing between trials. If not, then you should add more digits to the display, so you can average out the random error (instead of truncating the variation and losing access to error information).
3. The rules for propagating sig. figs. are relatively straightforward:

- (a) For multiplication and division, the sig. figs. of the result is limited by the operand which supplied the *smallest number* of sig. figs:

$$0.7 \times 6.626 = 5$$

- (b) For addition and subtraction, the sig. figs. of the result is limited by the number whose last sig. fig. is *farthest from zero*. There can be no digits smaller than the smallest decimal place of this limiting number.

$$6180.02517 + 2.1 \times 10^{-1} = 6180.24$$

- (c) If you are still confused, there is an excellent tutorial at Khan Academy (Google:Khan Academy sig figs). The 3rd and 4th video talk about the arithmetic rules.

4. Your final result must use the correct number of sig. figs. But you should **always** write down two or three *more* digits for intermediate calculations. This makes your final result less susceptible to rounding error.

1.4 Fit lines (a.k.a. trend lines)

The main quantitative tool of this course is the fit line. This is because fit lines are superior to simple averaging, which we can see in an experiment to verify Hooke's law

$$F = -kx . \tag{6}$$

To both verify this model and determine a spring's k , we can take three trials of the force required to stretch a spring to three different extensions. Table 1 calculates k for each configuration, then averages these to extract the spring constant (20.0 N/m). Fig. 2 determines the spring constant from the slope of its linear fit line (16.0 N/m).

x (cm)	Average F (N)	k
3.0	-0.618	0.206
5.5	-1.231	0.224
3.0	-3.492	0.170
	Average k	0.200

Table 1: The simple average

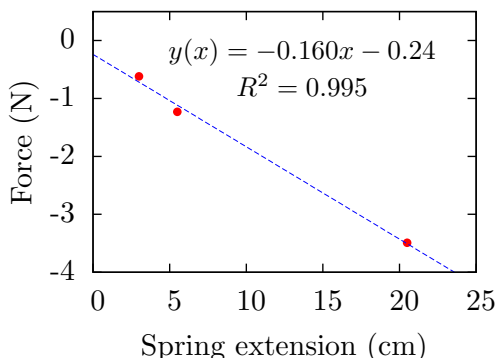


Figure 2: The fit line

These different methods produce spring constants which are about 20% different. What gives?

First, notice how two of the configurations are bunched together, while the other is off by itself. Since the solitary data point is much farther from the origin, it has more “leverage” to influence the slope of the fit line — it prefers a much shallower slope than the bunched data (which you can see by drawing a line between the bunched pair). In the simple average, each data point is blindly given equal influence, giving much more weight to the bunched pair.

Additionally, the linear fit has a non-zero y -intercept (which, if the model is correct, indicates a systematic measurement error in either F , x , or both). The simple average cannot extract this information, because F/x forces the y -intercept to be zero. This embeds any systematic measurement error into the slope, causing it to be steeper (or shallower) than it ought to be. For instance, note that *all three* k in Table 1 are larger than the slope of Fig. 2.

1.4.1 Show the fit line equation

Since science is quantitative, there is no purpose in showing a graph unless you plan on doing something numerical with it. In this course, that means adding a **linear best-fit line**. But since the only purpose of a fit line is to quantify an obvious trend in the data, there is no point in adding a fit line unless you **show the fit line equation** on the graph.

1. Make sure that when you create the fit line, you **do not** constrain the y -intercept to be zero. The y -intercept is a great way to test for systematic error, and it should never be exactly zero. Furthermore, a constrained fit will be worse (a smaller R^2), and the slope can be way off.
2. If your y -intercept is exactly zero, even though you did not constrain it to zero, you are *doing something wrong* (see Sec. 2.3.1).
3. Remember that your spreadsheet program doesn't know what units you are working with, so it will not display units in your fit line equation. Chances are, though, that your fit line is fitting data with units. When you discuss your results, don't forget to add those units back in.
4. Make sure your fit line equation shows the correct number of sig. figs.

1.4.2 Show the R^2

If you show your fit line equation, you might as well show the R^2 . This will allow you to quantify (and discuss) the strength of the fit.

1. The R^2 is **not** the exact numerical accuracy of your results; $R^2 = 0.90$ does not mean 90% accurate (because an $R^2 \leq 0.90$ is actually pretty poor).

R^2 value	How well is the data fit by the best-fit line?
$R^2 = 1$	Impossibly perfect (see #2 below)
$0.99 \leq R^2 < 1$	Excellent (ideal)
$0.98 \leq R^2 < 0.99$	Good
$0.95 \leq R^2 < 0.98$	OK
$0.90 \leq R^2 < 0.95$	Poor
$R^2 < 0.90$	Bad (data not very linear)

Table 2: Significance of a fit line's R^2 value

2. $R^2 = 1$ is listed as impossible because *science is not perfect*. There is no such thing as a perfect measuring device, so there are always random measurement errors.

- (a) If your $R^2 = 1$, you may be doing something wrong (plotting the data incorrectly); but it also could be *very close* to 1, and Excel is rounding it. You need to right-click on the fit line equation box to format it, then tell the program to display more digits. Keep displaying digits until the $R^2 < 1$ (e.g. 0.99997). This is for my purposes, so I can know that you are plotting your data correctly (and its OK if that means you're showing too many sig. figs. in your slope).
- (b) If the R^2 never deviates from 1, even after you added a bunch of digits, you are either plotting your data incorrectly (see Sec. 2.3.1), or you didn't take your data with enough sig. figs. in the lab (see Sec. 1.3).

2 Graphing

In order to get a good fit line, with a useful slope and y -intercept, you must be able to graph your data correctly. This section will address some common graphing mistakes.

2.1 Scatter plots only!

In this course, all of your graphs should be **scatter** plots, since we will never be measuring the kind of information that will benefit from a bar or pie graph. If you think another type of graph would be useful, ask me about it beforehand.

1. The **only lines** on your scatter plot should be linear best-fit lines that you've manually added. Don't use the scatter plot with the "connect-the-dots" lines, or the nice smooth Bezier curve — these lines are rubbish, not science.
2. The **absolute worst** graph you could give me is a smooth line connecting the data points, but without the data points themselves and no fit line or equation. This graph has nothing to offer (and I've seen it often enough to mention).

2.2 Getting the right axes

It is important to correctly assign your data to the horizontal and vertical axes. If you mistakenly swap the axes, your slope will be inverted (instead of 100, you'll get 0.01).

1. When you are instructed to make a plot of "**A vs. B**", **A** needs to go on the vertical axis (y -axis) and **B** on the horizontal axis (x -axis).
2. If you are fitting you data to a linear model ($y = m x + b$) to demonstrate Hooke's law ($F = -k x$), the vertical/ y -axis should plot force (F), and the horizontal/ x -axis should plot displacement (x). That way, the slope $m = -k$.
3. By default in Excel, if you highlight two adjacent columns and insert a plot, the data in the **left column** will be plotted on the x -axis, and the data in the **right column** will be plotted on the y -axis
 - (a) There is **one** exception to this rule: If there are only two data points (4 cells), then Excel will automatically select the **top row** for the x -axis, and the **bottom row** for the y -axis. However, since you should never plot a graph with only two data points (as it will always show a perfect fit), this shouldn't be a issue.

2.3 Nothing graphs

Too often I receive graphs which have fit lines, and show me their fit line equations and R^2 , but which have no informational value. Such graphs will not earn you any points. If you find yourself with one of these nothing graphs, and don't know how to proceed, ask me for help!

2.3.1 Plotting data versus itself

Often, you are tasked with comparing theory to experiment. For example, let's say you are asked to plot a graph of **Force vs. Acceleration**.

This graph needs to depict theory vs. experiment, so your experimentally measured acceleration should be plotted on the x -axis, and your theoretically calculated force on the y -axis. The slope of the resulting fit line will give you the mass of the accelerating object. However, the theoretically calculated force **can't be calculated** the easy way, by simply multiplying the experimental acceleration by the mass. This would plot ma vs. a , which is equivalent to plotting a versus itself! This guarantees that the slope is exactly equal to the measured mass, the y -intercept is exactly 0, and the R^2 is exactly 1.

This is a **nothing graph**, because it has nothing to say. *Of course* you get a perfect slope, a y -intercept of 0 and $R^2 = 1$; you built them into the graph.

2.3.2 Plotting “trial number”

The order in which you took a trial is not important, which means that the “trial number” is completely arbitrary and non-physical. If the x -axis of your graph seems to have integer labels (1, 2, 3, 4, 5, ...), then you are probably plotting trial number on x -axis. This is also a **nothing graph**, because you cannot fit a model to a graph with a non-physical axis.

In general, a “trial number” graph is created by only highlighting and plotting data from one column. Each graph must use data from at least **two columns**. If you don't know how to fix this problem, come see me during office hours.

2.4 Use data series to reduce clutter

Sometimes you will measure the response of a system while changing two independent properties. This means that there are two independent variables (those which control the configuration of the system), and one *dependent* variable (the measured response of the system, which you'll generally plot on the y -axis).

A simple example is a cart accelerating down an incline. You can vary the angle of the incline *or* the mass of the cart, then measure the cart's acceleration. If you choose to plot the cart mass on the x -axis, and acceleration on the y -axis, how will you depict the incline angle using only 2D plots? There are a couple of options:

1. Plot the second independent variable (e.g. the different incline angles) on separate graphs. This can mean a lot of graphs.
2. Plot the second independent variable using multiple data series on the same graph.
 - (a) After you plot the first set of data (e.g. the 5 degree incline), right-click on the graph and select “Add Data Series”.

- (b) You can add as many data series as you want, as long as they are distinguishable (their data points cannot obscure each other). Make sure they use different icons (diamonds versus circles), and are properly labeled in the key.
- (c) Each series needs its own best-fit line. Since you need to show the fit line equation for each, make sure it's clear which equation goes with which trend line. The easiest way is to print out the final lab report, then draw connecting lines by hand.

2.5 Truncation!

Let's say you want to accelerate a cart down an inclined track. The cart can carry various loads, and you want to determine if the total loaded mass (M) of the cart affects the travel time (T) to the bottom of the incline. You will observe two basic scenarios: (i) the mass of the cart affects the travel time, so a graph of T vs. M has a clear slope or (ii) the mass of the cart doesn't matter, T vs. M is flat.

We can examine these scenarios in Fig. 3; one system shows a clear response, and the other does not. Or does it? Strangely enough, both graphs have the *exact same* fit line. What's going on here?

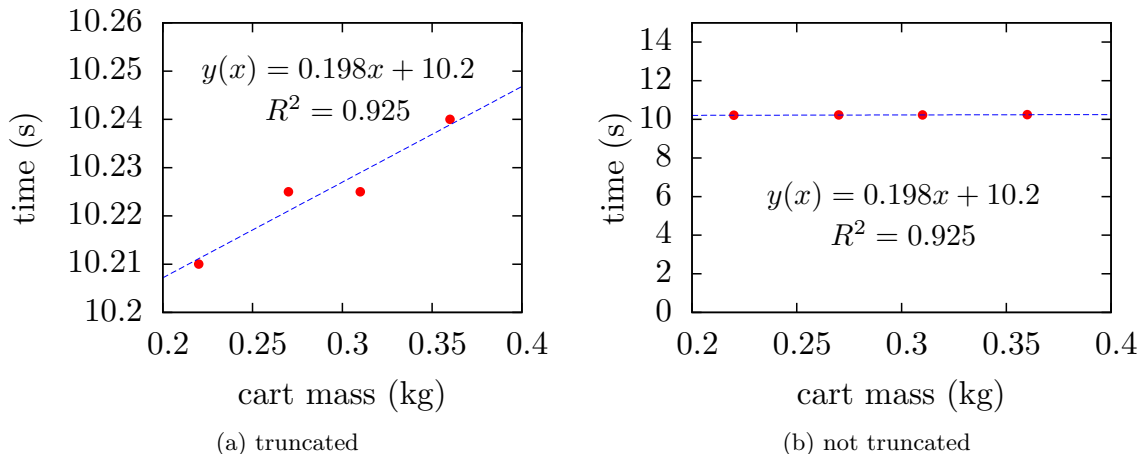


Figure 3: Two very different looking graphs

Actually, not only do both graphs have the exact same fit line, they depict the *exact same* data. The difference is in their y -axis; Fig. 3a zooms in *a lot*, so much that you can see the data's small fluctuations, while Fig. 3b gives the fluctuations some perspective by showing you $y = 0$. Truncating the y -axis can be dangerous; it can create the illusion of false trends, especially when there are only a handful of configurations (and it is actually quite probable that random fluctuations in flat data will create a phantom trend). Of course, the slope of the fit line gives us some idea that the data is actually very flat, since it is **100 times** smaller than the y -intercept.

1. In order to prevent these kind of mistakes, it is always important to show **zero** on the axis which shows your dependent variable (the one you don't control, which is generally the y -axis). Since Excel automatically sets the axis range, you may need to correct it.

- (a) To make an axis show zero, click on the axis until it is wrapped in a dotted box, then right-click on the box and select “Format Axis”. Once inside the formatting dialog box, change the minimum to zero (or the maximum if your data is negative).
2. You don’t need to show zero on the axis of your **independent variable**, since you control the independent variable, and should be measuring the system in sufficiently different states that the measurements aren’t exceedingly close to one another.