Good to the last drop: fully utilizing a *pp* collision's correlated information with the QCD power spectrum

Keith Pedersen
(kpeders1@hawk.iit.edu)
with
Zack Sullivan (Zack.Sullivan@iit.edu)

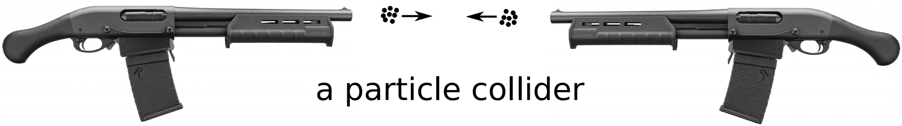
ILLINOIS INSTITUTE OF TECHNOLOGY

P-25 theory seminar, Los Alamos National Laboratory, 22 Oct 2018

These slides are now available at `www.HEPguy.com`

Outline

1. **The search for new physics at the LHC**
   - All bumps are created equal (but some are *more equal*)
   - Basics of LHC proton physics
2. **Revisiting the QCD power spectrum**
   - high-luminosity $\Longrightarrow$ high-pileup
   - Using all available information
   - The power spectrum H_ℓ (e.g., Fox-Wolfram moments)
3. **Modification 1: Shape functions $\Longrightarrow$ collinear safety**
   - H_ℓ for basic QCD events
   - The angular resolution of a finite sample
4. **Modification 2: The Power jets model**
   - The expected H_ℓ distributions
   - Fitting a jet-like model to the H_ℓ observation
   - Pileup: a natural extension
   - H_ℓ for high energy **nuclear** physics



a particle collider

Scattering jargon

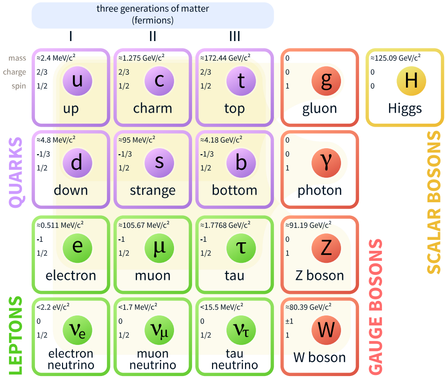
$$\sigma = \text{scattering cross section}$$

$$L = \text{collider luminosity}$$

$$L_{\text{int}} = \int L\,dt \qquad \text{(sample size)}$$

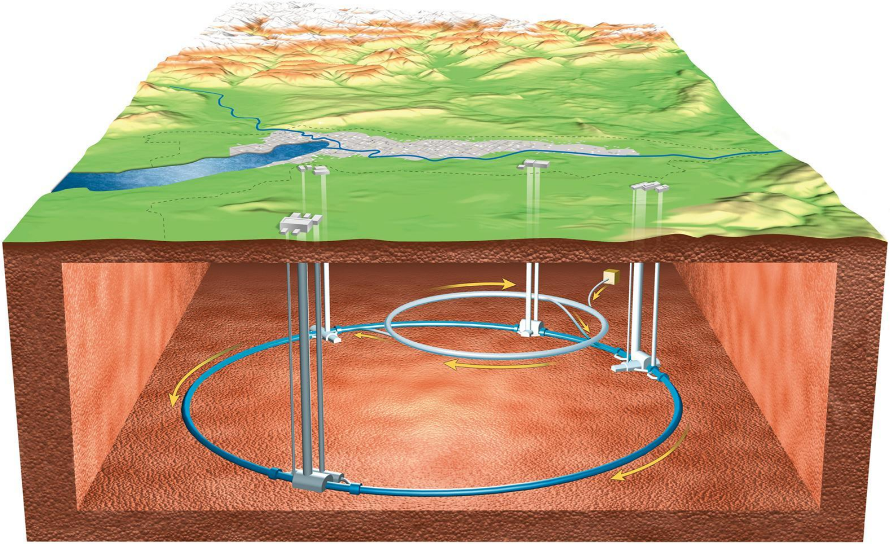
$$\text{Ex}\,(\text{collisions}) = \sigma(\text{cm}^2)\; L_{\text{int}}(\text{cm}^{-2})$$

Standard Model of Elementary Particles

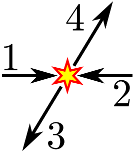


The Large Hadron Collider (LHC)

The Large Hadron Collider (LHC)



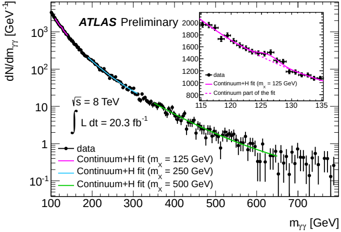
A tale of two bumps



$$p_1 p_2 \to X \to p_3 p_4$$

$$m_X = \sqrt{(p_3 + p_4)^2}$$

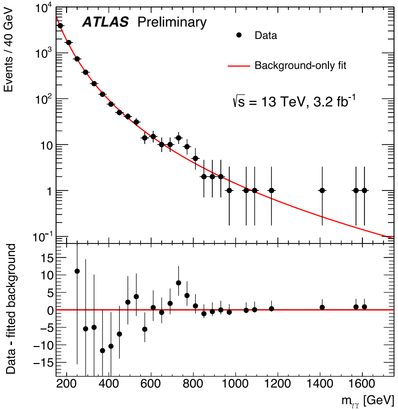
At $\sqrt{S} = 8\,\textbf{TeV}$, the LHC saw the Higgs boson at $m_{\gamma\gamma} = 125$ GeV



Nucl.Part.Phys.Proc. **273–275** (2016) 2460-2462.

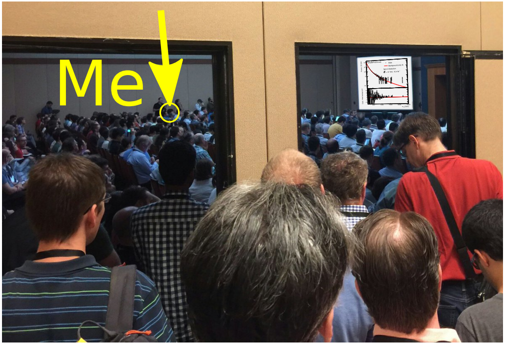
In 2015, first data at $\sqrt{S} = 13\,\textbf{TeV}$ saw excess in $m_{\gamma\gamma} \approx 750$ GeV.

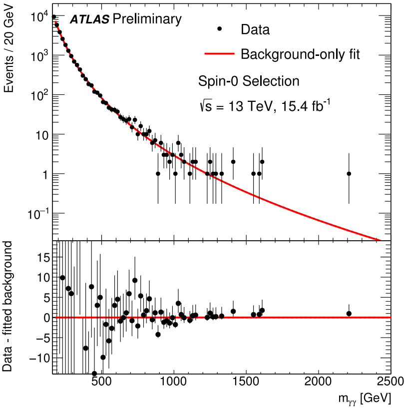
A new particle?



JHEP **1609** (2016) 001

Fall of the 750 GeV excess



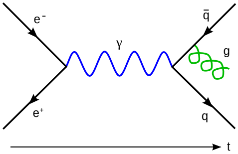


By Aug 5, 2016, with 5 × **more** data at $\sqrt{S} = 13$ TeV, the bump was gone.

The high-energy and high-luminosity frontier will face harder problems than statistical anomalies!.

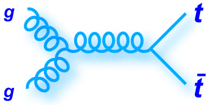
Collider energy and invariant mass

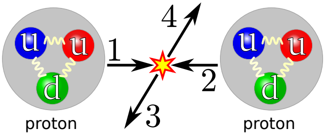
electron-positron collider



- $e^+e^- \to q\bar{q}g$
- $\boldsymbol{P}_1 = E_{\text{beam}}[1, +\hat{z}]$
- $\sqrt{S} = \sqrt{(\boldsymbol{P}_1 + \boldsymbol{P}_2)^2} = 2E_{\text{beam}}$

A proton collider is really a parton collider

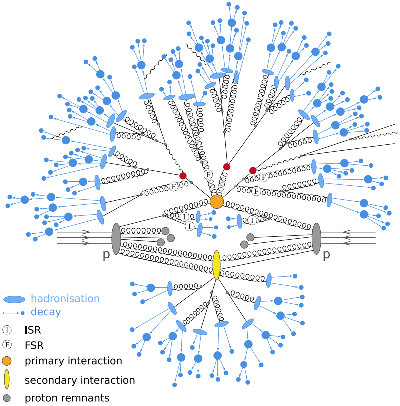


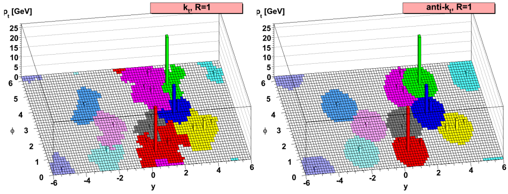


- not $pp \to q\bar{q}$ but:

  $q\bar{q} \to q'\bar{q}'$, $qg \to qg$, $gg \to q\bar{q}$
- $\boldsymbol{p}_1 = x_1 \boldsymbol{P}_1$
- $\sqrt{s} = \sqrt{(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2} = 2\sqrt{x_1 x_2}\, E_{\text{beam}}$

New physics is wrapped in QCD





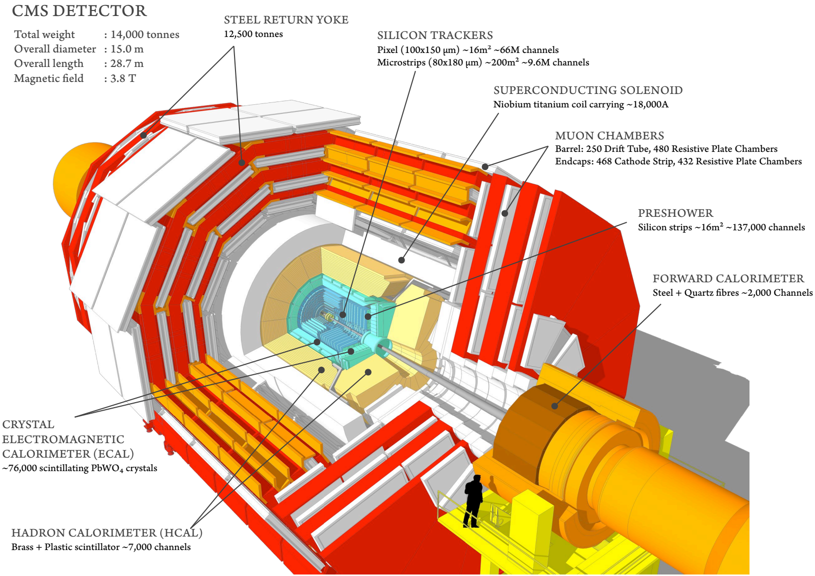
QCD has **asymptotic freedom**; hard scatter $\mapsto$ busy final state.

- Initial-state radiation.
- Final-state radiation from quarks/gluons creates **jets**;
  - **jet-parton duality**
- Confinement . . . colored particles must hadronize.

Reconstructing quark/gluon jets requires a **jet definition**.

- k_T jets *rewind* QCD shower.
- anti-k_T less sensitive to soft physics; popular at LHC.

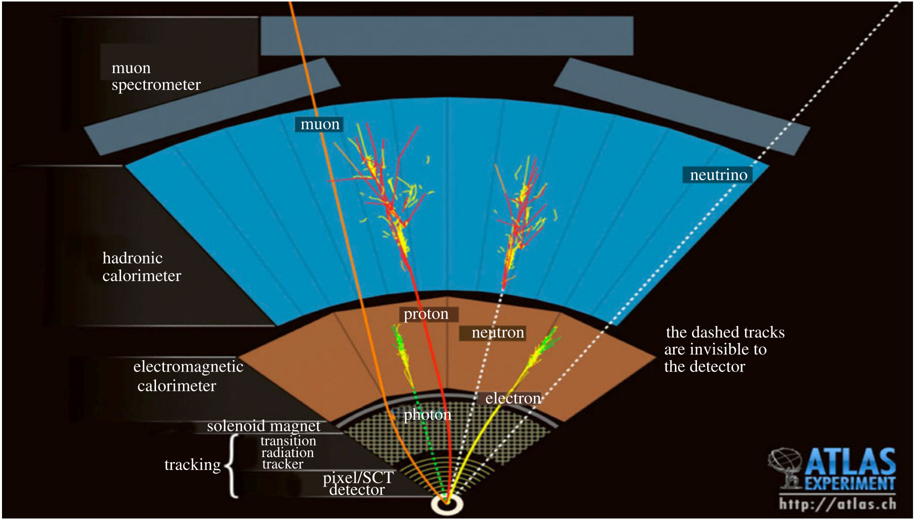
An LHC detector



What a detector sees

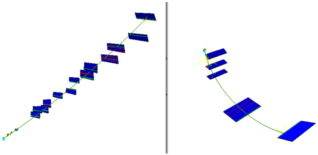
charged $\mapsto$ **tracks**

neutral $\mapsto$ **towers**

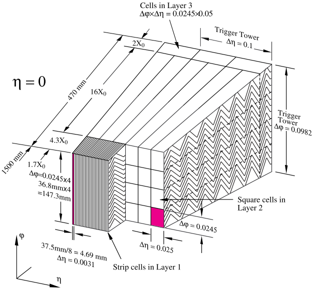


Physics objects: tracks and towers

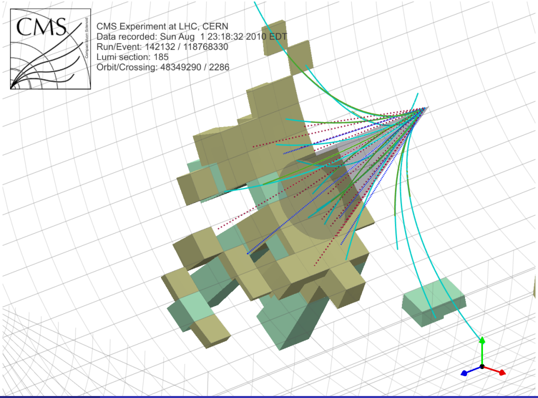
tracks



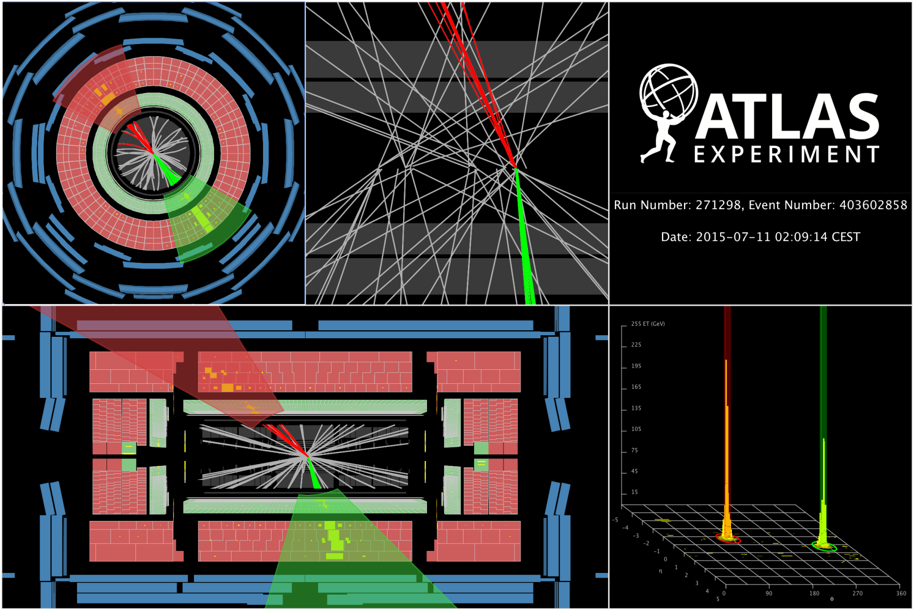
towers



- **Neutral tower**: track energy subtracted from tower that was struck.
- **massless** tracks and neutral towers are clustered into **massive** jets.



A two-jet event with $\sqrt{s} = 3.25$ TeV

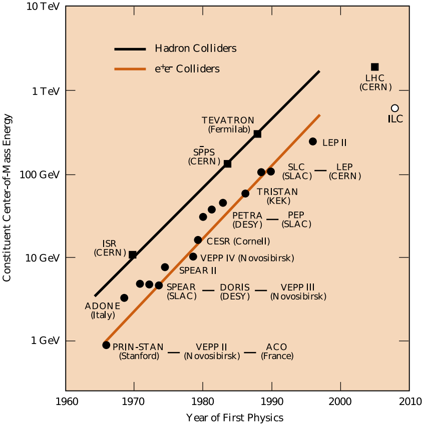


Outline

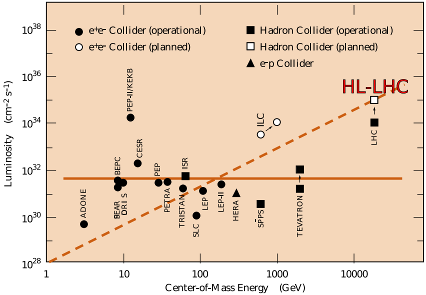
1. The search for new physics at the LHC
   - All bumps are created equal (but some are *more equal*)
   - Basics of LHC proton physics
2. **Revisiting the QCD power spectrum**
   - high-luminosity $\Longrightarrow$ high-pileup
   - Using all available information
   - The power spectrum H_ℓ (e.g., Fox-Wolfram moments)
3. Modification 1: Shape functions $\Longrightarrow$ collinear safety
   - H_ℓ for basic QCD events
   - The angular resolution of a finite sample
4. Modification 2: The Power jets model
   - The expected H_ℓ distributions
   - Fitting a jet-like model to the H_ℓ observation
   - Pileup: a natural extension
   - H_ℓ for high energy **nuclear** physics

How to find new physics:

1. Increase collider energy $\sqrt{S}$.
2. Increase luminosity L_{int}.



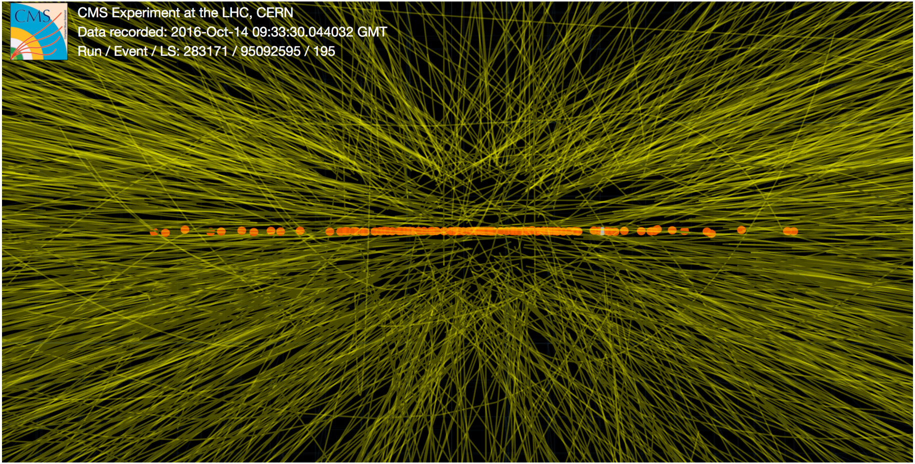
- Higher $\sqrt{S}$ unlocks new physics.
- Higher L creates more events (better stats, more precision).



A caveat:

- More events $\mapsto$ more pileup.

Pileup is here to stay



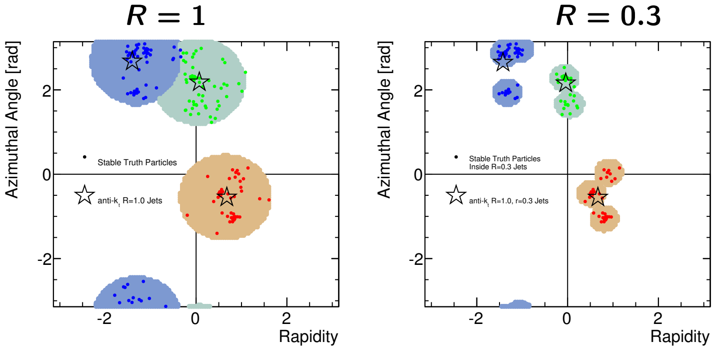
- Most pileup from other vertex — **charged** pileup is largely reducible.
- The LHC is currently averaging **40 pileup events** per hard scatter!
- The HL-LHC is expected to average $\mathcal{O}(200)$!

Pileup in anti-k_T jets

anti-k_T jets use one correlation at a time; find the smallest "distance"

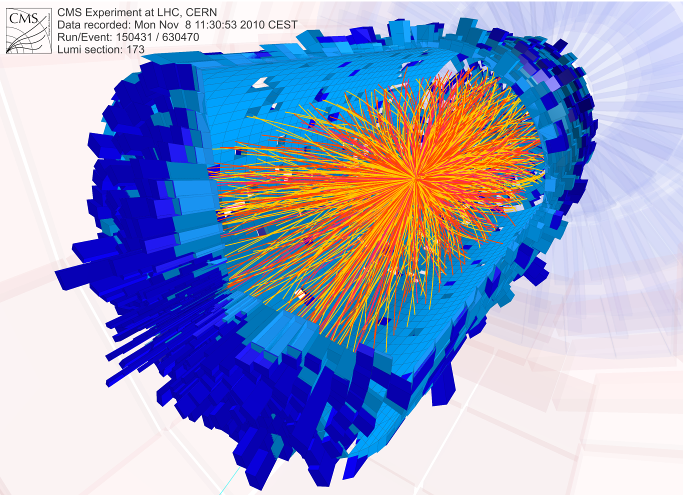
$$d_i^2 = p_{T,i}^{-2}$$ cluster becomes a jet

$$d_{ij}^2 = \min(p_{T,i}^{-2}, p_{T,j}^{-2})\frac{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}{R^2}$$ merge two clusters

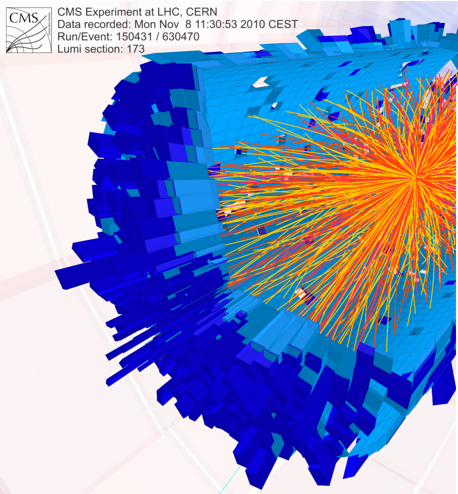


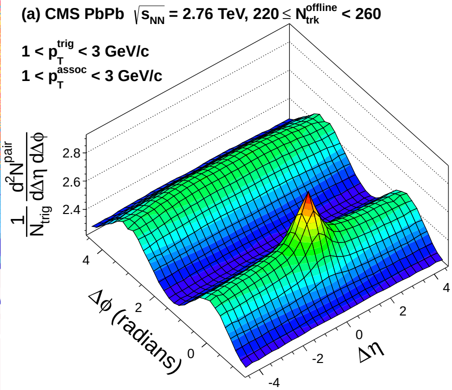
Eur.Phys.J. **C76** (2016) 581

Learning from heavy-ion collisions



Learning from heavy-ion collisions



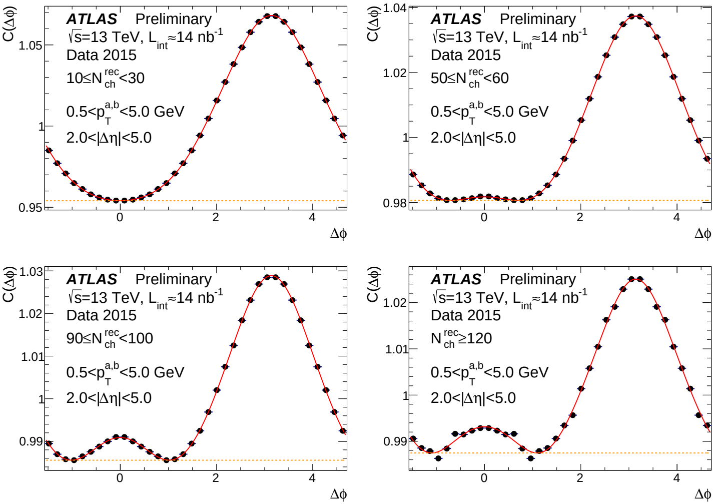


Phys.Lett. **B724** (2013) 213–240

The same-side ridge is attributed to collective flow of nuclear media.

Connecting lead-lead to proton-proton

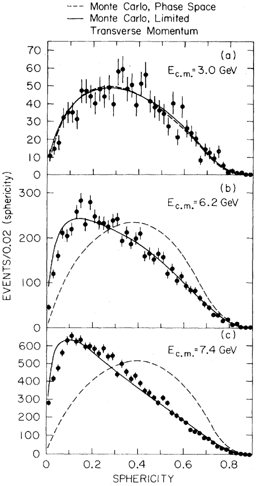
ATLAS-CONF-2015-027



Why is the same-side correlation seen in high-multiplicity *pp* collisions?

Event shape variables

Sphericity



Phys.Rev.Lett. **35** (1975) 1609–1612

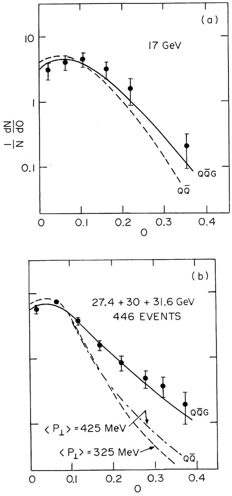
$\Longleftarrow$ 2-jet structure ($e^+e^- \to q\bar{q}$);
first seen with Sphericity.

3-jet structure $\Longrightarrow$
($e^+e^- \to q\bar{q}g$);
first seen with Oblateness.

Event shape variables:

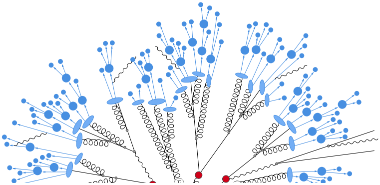
- Condense each event to a single number.
- Shape curves from many events.

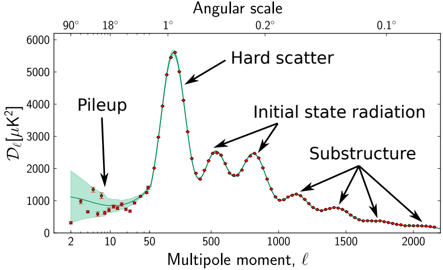
Oblateness



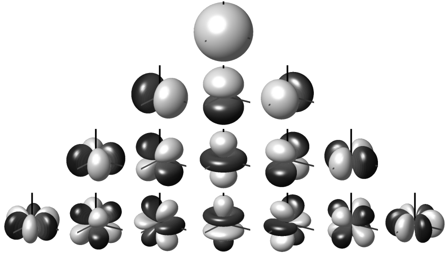
Phys.Rev.Lett. **43** (1979) 830

The power spectrum of QCD radiation





$$S_\ell \equiv \sum_{m=-l}^{l} |E_\ell^m|^2$$



Spherical harmonics $Y_\ell^m(\theta, \phi)$

- degree ℓ — number of lobes.
- order m — lobe orientation.

$$E(\hat{r}) = \sum_i E_i \, \delta(\hat{r} - \hat{p}_i)$$

$$E_\ell^m = \int_\Omega \mathrm{d}\Omega \, Y_\ell^{m*}(\hat{r}) \, E(\hat{r}) \, .$$

The dimensionless power spectrum H_l

A dimensionless power spectrum scales out total detected energy E_{tot}

$$H_\ell \equiv \frac{1}{2\ell+1}\frac{\sum_m |E_\ell^m|^2}{E_{\text{tot}}^2} = \frac{1}{4\pi}\int_\Omega \mathrm{d}\Omega \int_{\Omega'} \mathrm{d}\Omega' \rho(\hat{r})\rho(\hat{r}')P_\ell(\hat{r}\cdot\hat{r}')$$

$$H_0 = 1 \qquad 0 \le H_\ell \le 1 \qquad \xi_{\text{res}} = \frac{2\pi}{\ell}$$

$$\rho(\hat{r}) = \sum_i f_i\,\delta(\hat{r}-\hat{p}_i) \qquad f_i \equiv \frac{E_i}{E_{\text{tot}}} \qquad \xi_{ij} \equiv \hat{p}_i\cdot\hat{p}_j$$

Fox-Wolfram event shape — energy fraction — inter-particle angle

---

★ $$H_\ell = \sum_{i,j} f_i\, f_j\, P_\ell(\cos\xi_{ij}) = \langle f|\, P_\ell\Big(\,|\hat{p}\rangle\cdot\langle\hat{p}|\,\Big)\,|f\rangle$$

Fox and Wolfram, Phys. Rev. Lett. **41** (1978) 1581

Infrared and collinear safety of H_ℓ

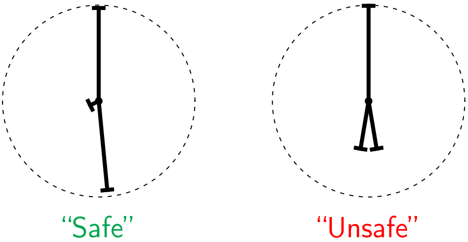
$$H_\ell = \sum_{i,j} f_i\, f_j\, P_\ell(\cos\xi_{ij})$$

How is H_ℓ affected when a particle radiates ($a \to b\,c$)?

- **Infrared**: a soft particle ($f \ll 1$) has minimal weight in the H_ℓ sum.
- **Collinear**: daughters are not soft; creates small-angle correlations.

The Fox-Wolfram power spectrum is infrared safe, but collinear unsafe.

- Ignore H_ℓ above ℓ_{max}?
- How to determine ℓ_{max}?
- **How much meaningful information exists in an N-particle final state?**

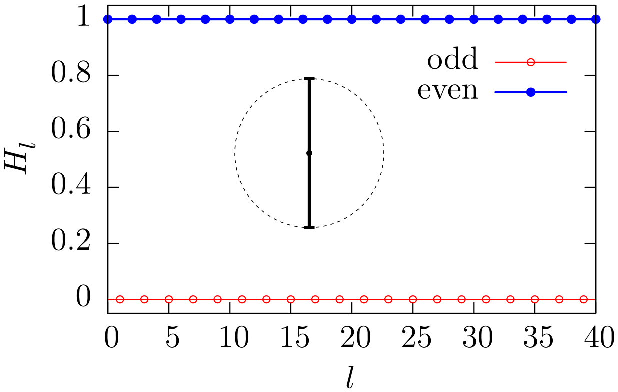


Outline

1. The search for new physics at the LHC
   - All bumps are created equal (but some are *more equal*)
   - Basics of LHC proton physics
2. Revisiting the QCD power spectrum
   - high-luminosity $\Longrightarrow$ high-pileup
   - Using all available information
   - The power spectrum H_ℓ (e.g., Fox-Wolfram moments)
3. Modification 1: Shape functions $\Longrightarrow$ collinear safety
   - H_ℓ for basic QCD events
   - The angular resolution of a finite sample
4. Modification 2: The Power jets model
   - The expected H_ℓ distributions
   - Fitting a jet-like model to the H_ℓ observation
   - Pileup: a natural extension
   - H_ℓ for high energy **nuclear** physics

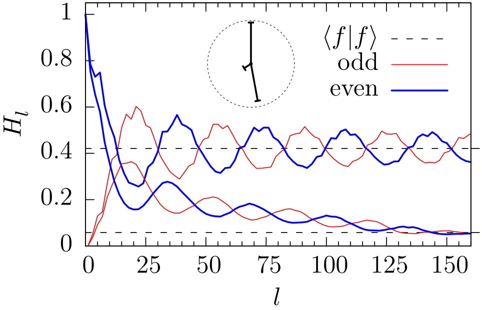
Every 2-particle event

A two-particle event in the CM frame is not just a large H_2 moment.



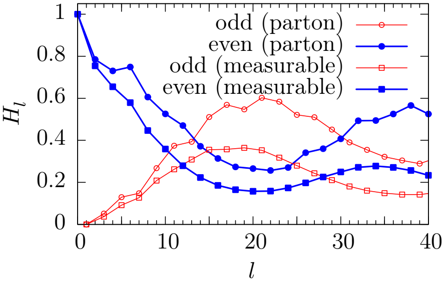
$$\rho(\hat{r}) = \delta(\hat{r} + \hat{z}) + \delta(\hat{r} - \hat{z}) = \sum_{\ell \in \text{even}} \sqrt{\frac{2\ell + 1}{4\pi}} Y_\ell^0(\hat{r})$$

A 2-jet-like event (truth level)

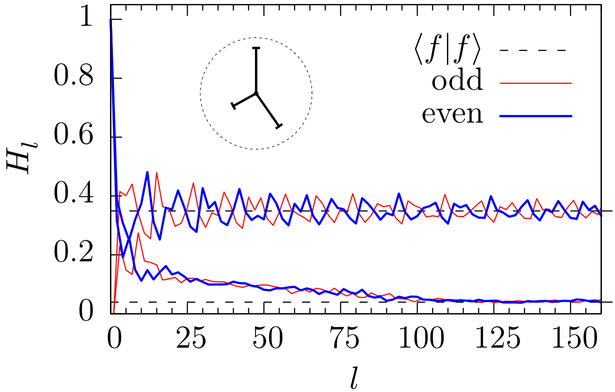


- No broad CMB-like shapes!
- $H_\ell \sim \langle f|f\rangle$
- Oscillation about $\langle f|f\rangle$ implies correlation between high-ℓ moments.

- H_2 is large; H_3 is small.
- Measurable particles only match originating partons at low ℓ.
  Jet structure matters!



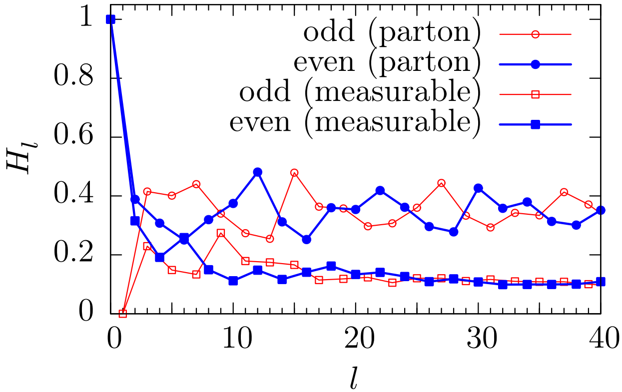
A 3-jet-like event (truth level)



Important features

- **H_ℓ rapidly oscillates**: ~~CMB~~
- **H_ℓ is unending**: $H_\ell \sim \langle f|f\rangle$
- **$N \neq n$**: N measurable particles don't match n original partons; *jet structure matters*.

A 3-jet-like event (truth level)

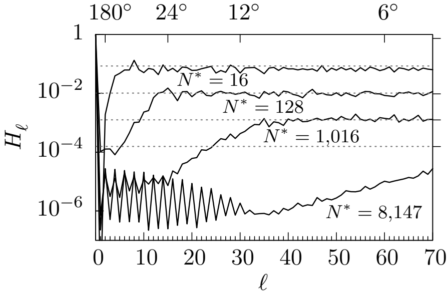


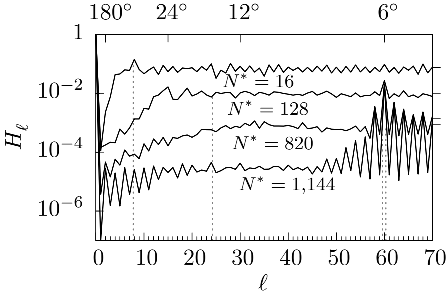
Important features

- **H_ℓ rapidly oscillates**: ~~CMB~~
- **H_ℓ is unending**: $H_\ell \sim \langle f | f \rangle$
- **$N \neq n$**: N measurable particles don't match n original partons; *jet structure matters.*

The multiplicity plateau and detector artifacts

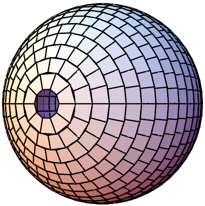
Track-only $\Longleftarrow$ $\left\{\text{Random isotropic } (\rho(\hat{r}) = \frac{1}{4\pi})\right\}$ $\Longrightarrow$ Tower-only





$$H_\ell = \langle f|\, P_\ell\Big(\,|\hat{p}\rangle \cdot \langle \hat{p}|\,\Big)\,|f\rangle = \langle f|f\rangle + \text{(inter-particle)}$$

$$\langle f|f\rangle \propto \frac{1}{N}\,; \qquad \langle f|f\rangle \geq \frac{1}{N}$$



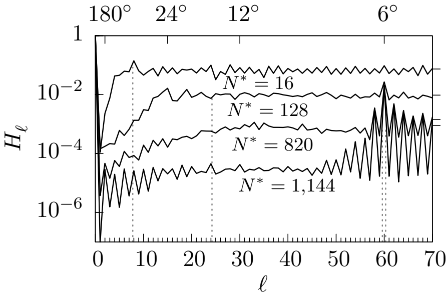
Multiplicity N limits angular resolution!

A sample's intrinsic angular resolution

A meaningful correlation must exceed the plateau at $\langle f|f\rangle \sim \frac{1}{N}$.

A **conservative** estimate of the sample's angular resolution $\xi_{\min}$:

1. Sort inter-particle angles ξ_{ij}.
2. Find the k smallest ξ_{ij} whose total weight $\sum f_i\, f_j \geq \langle f|f\rangle$.
3. $\xi_{\min} = \text{GeoMean}(k \text{ smallest } \xi_{ij})$.



Suppress small-angle correlations; shape functions $\Rightarrow$ extensive objects:

$$\rho(\hat{r}) = \cancel{\sum_i f_i\,\delta(\hat{r} - \hat{p}_i)} = \sum_i f_i\, h_i(\hat{r})$$

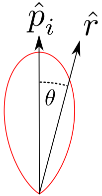
Natural resolution: kill correlations beyond the angular resolution $\xi_{\min}$.

Shape functions as low-pass filters

Natural resolution: kill correlations beyond ξ_{min} with shape functions:

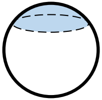
$$\rho(\hat{r}) = \cancel{\sum_i f_i\,\delta(\hat{r}-\hat{p}_i)} = \sum_i f_i\, h_i(\hat{r})$$

Tracks:
pseudo-normal in polar angle θ:



$$h(\theta) \approx C\,\exp\left(-\frac{\theta^2}{2\lambda^2}\right)$$

Towers:
spherical cap spanning each tower's solid angle Ω_{twr}.

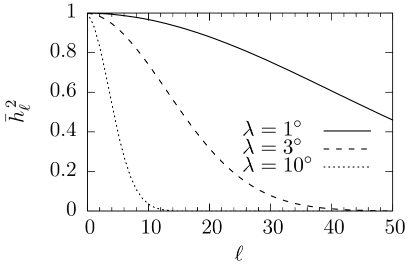


Adds shape coefficients $\bar{h}_\ell$ to H_ℓ:

$$H_\ell = \sum_{i,j} \bar{h}_{(i)\ell}\, \bar{h}_{(j)\ell} \underbrace{\left(f_i\, f_j\, P_\ell\left(\hat{p}_i\cdot\hat{p}_j\right)\right)}_{H_\ell \text{ of } \delta\text{-distribution}}$$

If all $\bar{h}_\ell$ have similar values:

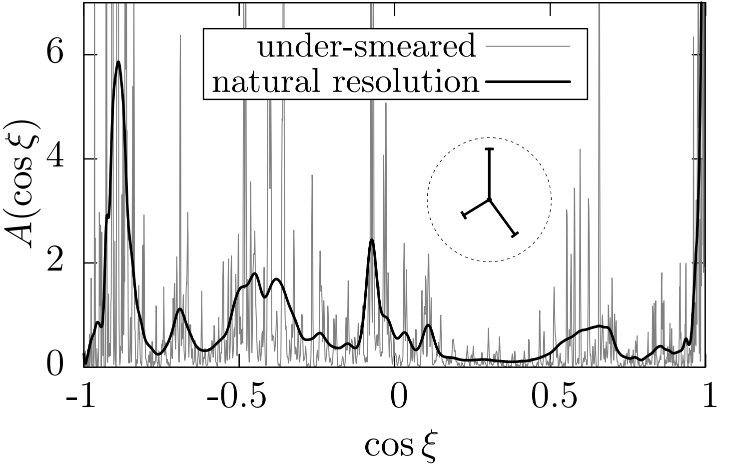
$$H_\ell \approx h_\ell^2\, H_\ell^{\delta-\text{particle}}$$



Shape functions restore collinear safety

Angular correlation function
(EEC for infinitesimal Ω)

$$A(\cos\xi) = \sum_{\ell}(2\ell+1)\, H_\ell\, P_\ell(\cos\xi)$$



Outline

1. The search for new physics at the LHC
   - All bumps are created equal (but some are *more equal*)
   - Basics of LHC proton physics
2. Revisiting the QCD power spectrum
   - high-luminosity $\Longrightarrow$ high-pileup
   - Using all available information
   - The power spectrum H_ℓ (e.g., Fox-Wolfram moments)
3. Modification 1: Shape functions $\Longrightarrow$ collinear safety
   - H_ℓ for basic QCD events
   - The angular resolution of a finite sample
4. Modification 2: The Power jets model
   - The expected H_ℓ distributions
   - Fitting a jet-like model to the H_ℓ observation
   - Pileup: a natural extension
   - H_ℓ for high energy **nuclear** physics

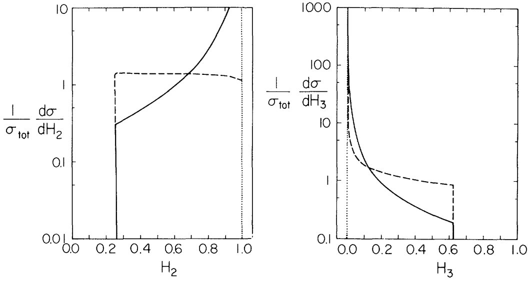
The expected H_ℓ distributions

Fox and Wolfram defined H_ℓ to differentiate two final states:

- $e^+e^- \to \gamma \to q\bar{q}g$ .... generic QCD.
- $e^+e^- \to X \to ggg$ .... a new, heavy resonance.

Integrate over $\frac{\mathrm{d}\sigma}{\sigma \prod_i \mathrm{d}p_i^\mu}$ to generate probability distributions $f(H_\ell)$:

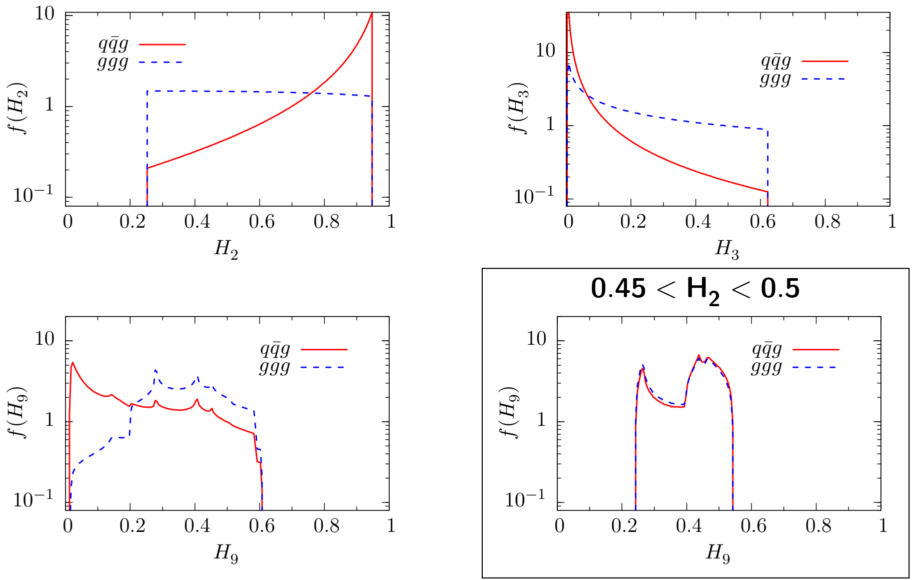
Phys. Rev. Lett 41 (1978) 1581



$q\bar{q}$ (dotted), $q\bar{q}g$ (solid), $X \to ggg$ (dashed)

- QCD radiation fluctuates event-to-event:
  - Angular resolution ξ_{min} depends on multiplicity N.
  - High-ℓ moments depend on jet shape ($N \neq n$).
- $f(H_\ell)$ for different ℓ are not independent!

Fox-Wolfram $f(H_\ell)$ are not independent.



The power jets fit

observable power spectrum

$$\rho(\hat{r})_{\text{obs}} = \sum_{i=1}^{N} f_i\, h_i(\hat{r})$$

$$\Downarrow$$

$$H_\ell^{\text{obs}}$$

$N \gg n$

$$\chi_\ell = H_\ell^{\text{fit}} - H_\ell^{\text{obs}}$$

n-prong power spectrum

$$\rho(\hat{r})_{\text{fit}} = \sum_{j=1}^{n} f_j\, h_j(\hat{r})$$

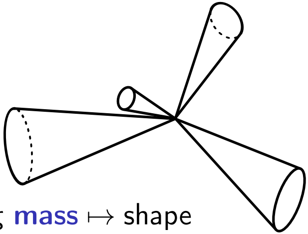
$$\Downarrow$$

$$H_\ell^{\text{fit}}$$

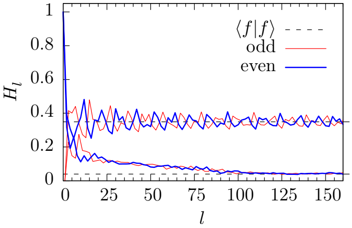
---

prongs $\Rightarrow$ **hard** radiation

prong shape $h_j(\hat{r}) \Rightarrow$ **soft** radiation



prong **mass** $\mapsto$ shape

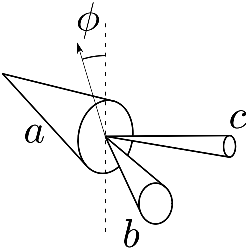


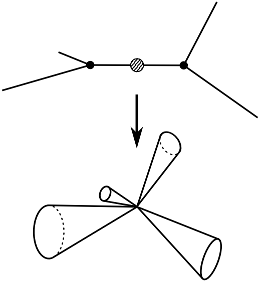
The power jets model

Describe hard QCD radiation with a binary splitting tree ($a \to b\,c$).

$$\boldsymbol{p}_a = \boldsymbol{p}_b + \boldsymbol{p}_c$$

Four degrees of freedom per splitting node ($\boldsymbol{p}_b = [E_b,\, \vec{p}_b\,]$).





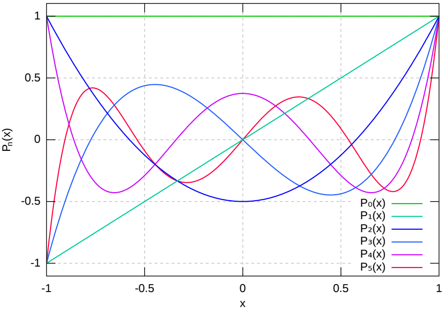
Prong shape $h_j(\hat{r})$ needs physical basis (not pseudo-normal a priori).

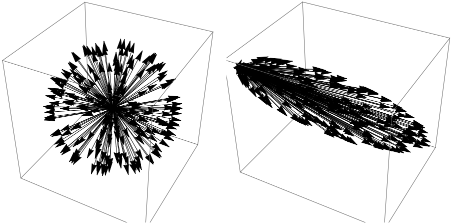
Prong shape functions

Prong shape in CM frame — azimuthally symmetric Legendre series:

$$h_{\mathrm{CM}}(\hat{r}) = \frac{1}{2} + \sum_{\ell=2}^{\infty} c_\ell \, P_\ell(\hat{r}\cdot\hat{p}) \xrightarrow[\text{lab frame}]{\textbf{Boost}\text{ to}} h(\hat{r}) \xrightarrow[\text{coefficient}]{\text{Calculate}} \bar{h}_\ell$$

- **Boost** determined from p^μ.
- c_ℓ constrained by $h_{\mathrm{CM}}(\hat{r}) \geq 0$.

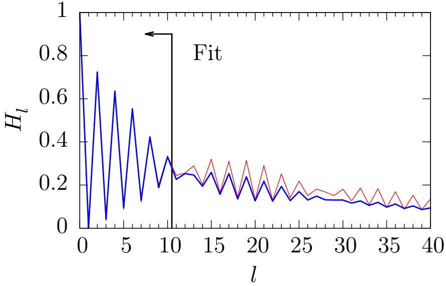




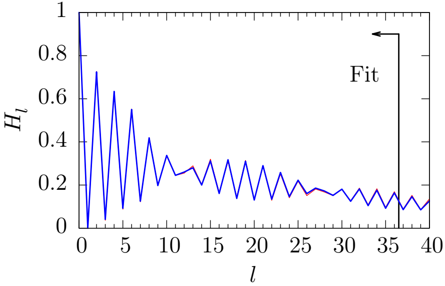
We restrict our initial studies to scalar ($J = 0$) CM shape.

Fitting a 2-jet-like event

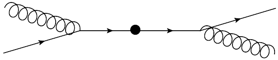
3-prong

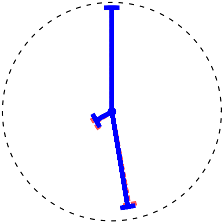


4-prong

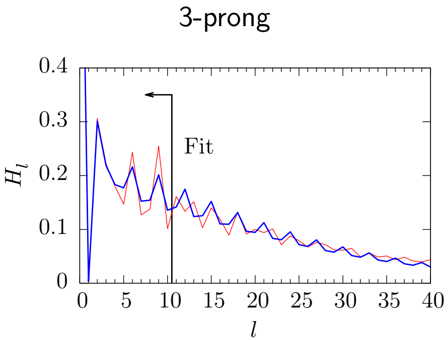


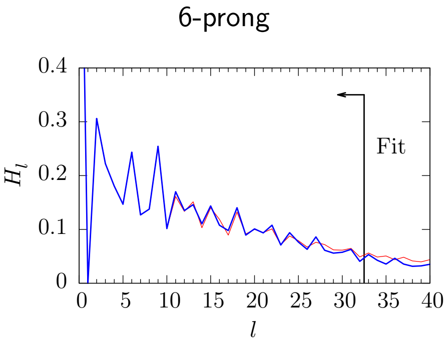
The 3-prong model doesn't match $l > 10$ (36°); need another prong.



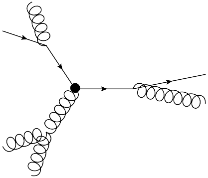


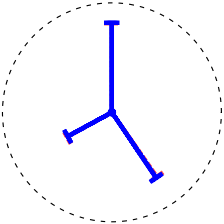
Fitting a 3-jet-like event



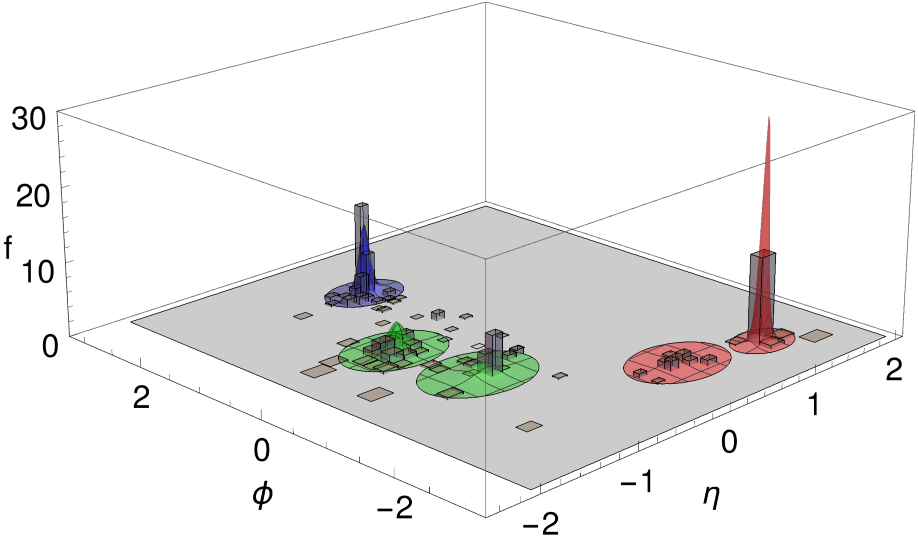


For a 3-jet-like event — 6 prongs:

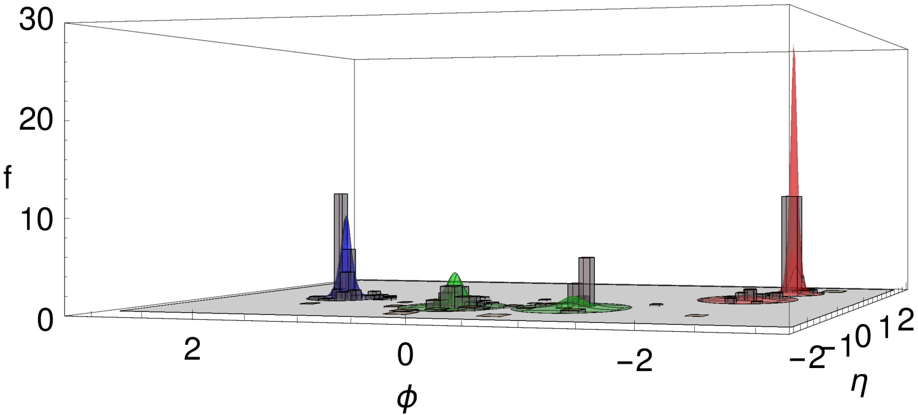




Jets without boundaries



Jets without boundaries



- **No fixed radius R** . . . narrow and fat topologies can coexist.
- No exclusive constituents . . . boundary particles shared.

Power jets provide superb reconstruction

Table : Reconstructed 3-jet kinematics for the 2-jet-like event.

(GeV)	E_1	E_2	E_3
parton	190.1	172.8	37.00
power jets	190.4(0)	174.2(1)	35.52(8)
error	0.1%	0.7%	-4%

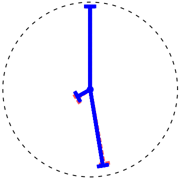
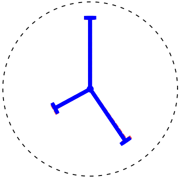


Table : Reconstructed 3-jet kinematics for the 3-jet-like event.

(GeV)	E_1	E_2	E_3
parton	163.0	143.5	93.56
power jets	162.0(1)	146.3(4)	91.68(4)
error	-0.6%	2.0%	-2.0%



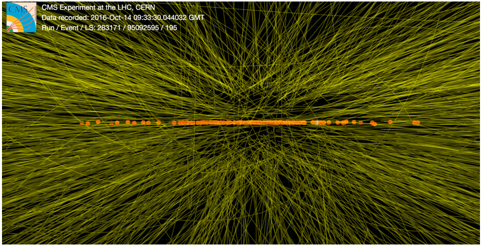
Pileup (soft QCD) is a global shape

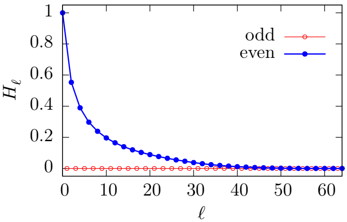
Add pileup to the event shape:

$$\rho(\hat{r}) = \rho(\hat{r})_{\text{hard}} + \rho(\hat{r})_{\text{PU}} = (1 - f_{\text{PU}}) \sum_j f_j \, h_{(j)}(\hat{r}) + f_{\text{PU}} \, h_{\text{PU}}(\hat{r})$$

$h_{\text{PU}}(\hat{r})$ can be **measured** from pileup-only events (lacking a hard scatter).

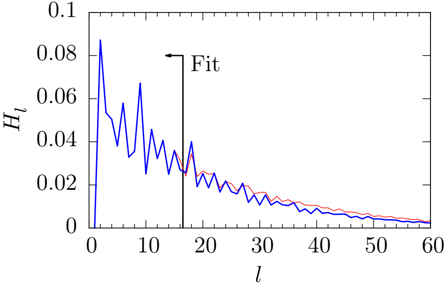
- Measure pileup H_ℓ directly; no soft-QCD model needed!
- Pileup-only events are abundant (min-bias)! LHC's trash $\rightarrow$ treasure.
- 1 free parameter; pileup energy fraction f_{PU}.



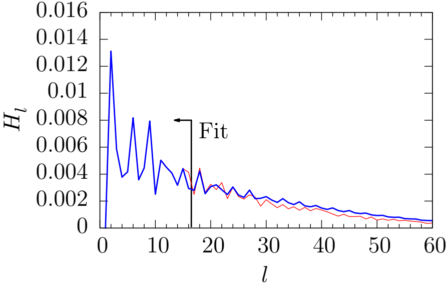


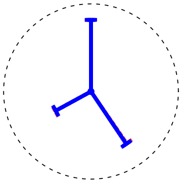
Using noise-noise correlations to see the signal

$S/N = 1 \quad (f_{\mathsf{PU}} = 0.5)$

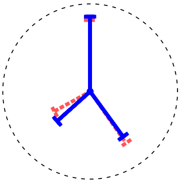


$S/N = 1/5 \quad (f_{\mathsf{PU}} = 0.8)$

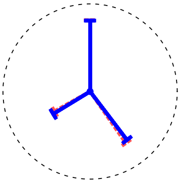




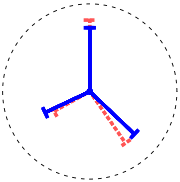
power jets



anti-k_T



power jets



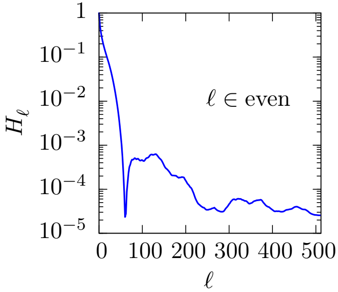
anti-k_T

Heavy-ion collisions

The power spectrum is naturally suited for **global shapes**:

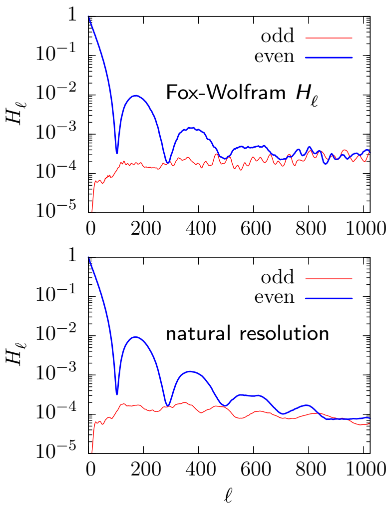
- Each local prong needs at least **four** free parameters.
- The **global shape** of *pp* pileup required only **one** parameter.

Power spectrum of *pp* pileup

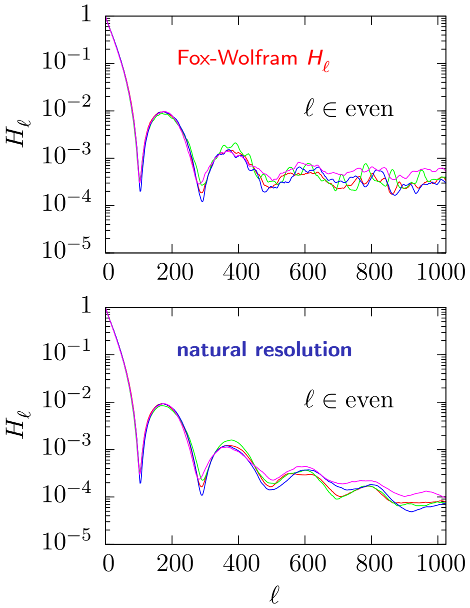


Pb-Pb collisions ⇒ **global shapes**:

Power spectrum of a Pb-Pb collision



See more by using less!



Five unrelated Pythia heavy-ion events (Pb-Pb; $\sqrt{S} = 2.76\,\mathrm{GeV}$).

- The raw H_ℓ (Fox-Wolfram) is sensitive to local fluctuations at high-ℓ.
- The **refined** power spectrum is far smoother:
  - Angular resolution ξ_{min}.
  - Smear tracks to ξ_{min} with pseudo-normal shape.
  - Towers use circular cap subtending Ω_{twr}.

A low-pass filter reveals **common structure**; exciting possibilities!

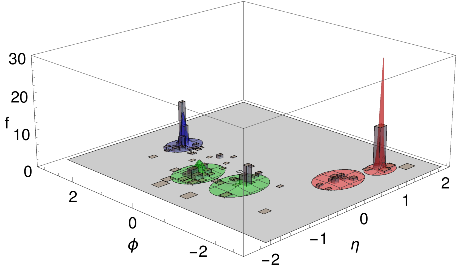
Fully utilizing global correlations

We modify the QCD power spectrum:

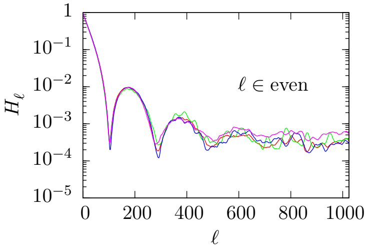
1. shape functions ⇒ low-pass filter.
2. Fit H_ℓ^{obs} to an n-prong model.

A simultaneous fit to all information:

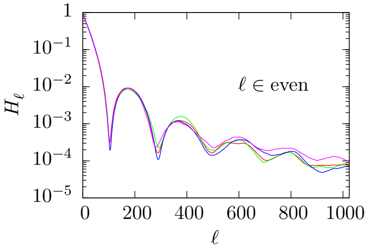
- Jets without boundaries.
- Pileup without subtraction.



What can the refined power spectrum tell us about nuclear physics?



natural ⇛ resolution



Thank you

Thank you for your attention!