Spherical harmonics for multiparticle final states

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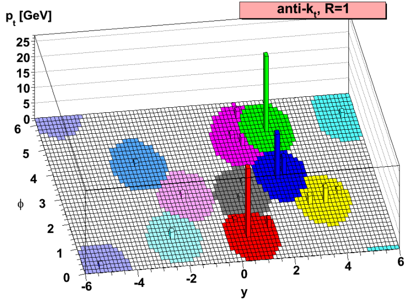
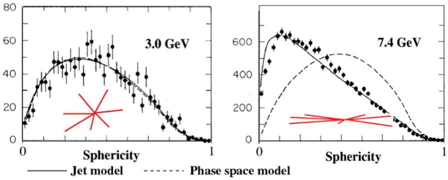
Outline

1. **The shape of QCD**
   - Can we probe QCD like the CMB power spectrum?
   - Can we suppress/identify pileup?
2. **A multipole expansion**
   - The power spectrum of multiparticle final states
   - Interpreting the power spectrum
3. **Jet shapes from the QCD power spectrum**
   - The H_l jet definition
   - Adding jet shapes
   - Sweeping away pileup

The ideal event shape variable





Jet clustering from N physics objects; at each step, the evolution is guided by only one of the N^2 two-particle correlations.

Event shape variables (sphericity) are more holistic, but tend to be 1-dim.

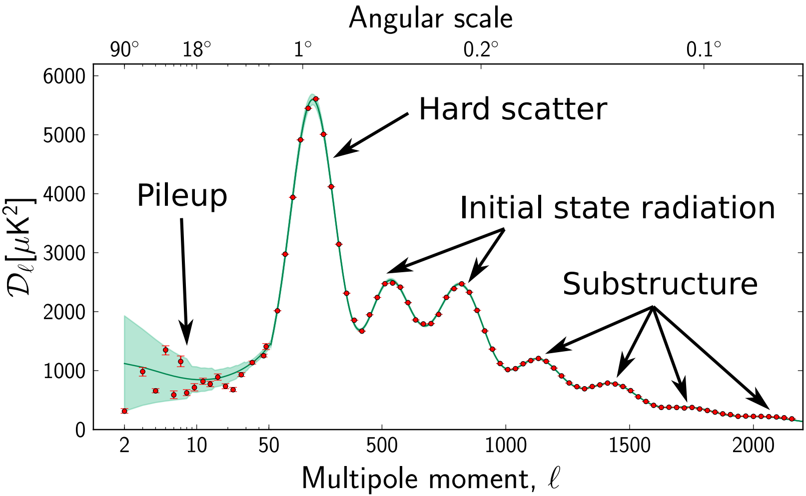
Ideally, a shape curve could describe a **single event**, and we could:

- Extract jet kinematics.
- Tag interesting signatures.
- Probe QCD at new scales.

A multipole expansion of E density?

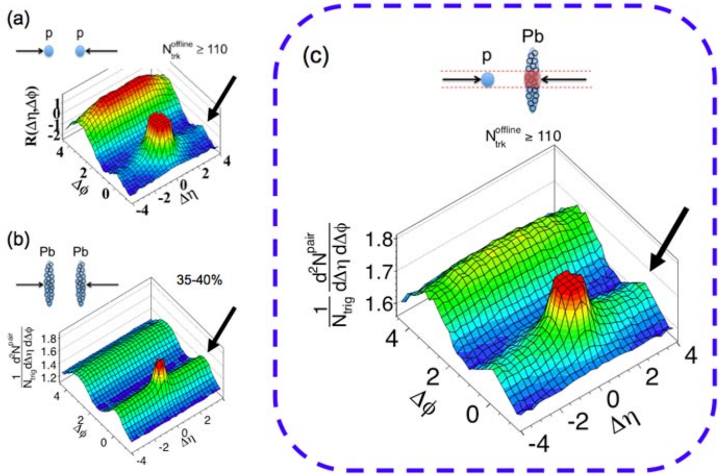
$$\rho_l^m = \int \mathrm{d}\Omega \, Y_l^{m*}(\theta,\phi)\, \rho(\theta,\phi)$$

Can we characterize final states like the CMB?



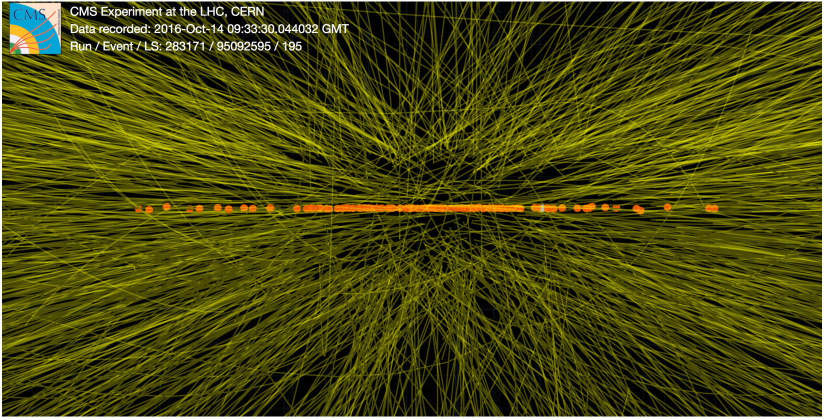
Can we identify broad shapes with known physics?

Can we study long distance correlations?



Can the ridge be **quark-gluon plasma** if we see it in *pp* collisions?

Can we identify/suppress pileup at the HL-LHC?



Pileup at the HL-LHC will be **intense**. We will need to:

- Remove pileup from jets and identify pileup-only jets.
- Distinguish **boosted top** from **QCD jets + pileup**.
- Find $W \to q\bar{q}$ for precision electroweak measurements.

Outline

The power spectrum H_l of a single event

Expand an event's energy distribution $\rho(\theta, \phi)$ into spherical harmonics

$$\rho_l^m = \int \mathrm{d}\Omega \, Y_l^{m*}(\theta, \phi) \, \rho(\theta, \phi) \quad \Longrightarrow \quad \boxed{H_l = \frac{1}{E_{\mathrm{tot}}^2} \frac{4\pi}{2l+1} \sum_{m=-l}^{l} |\rho_l^m|^2}$$

We detect a finite sampling of N discrete particles

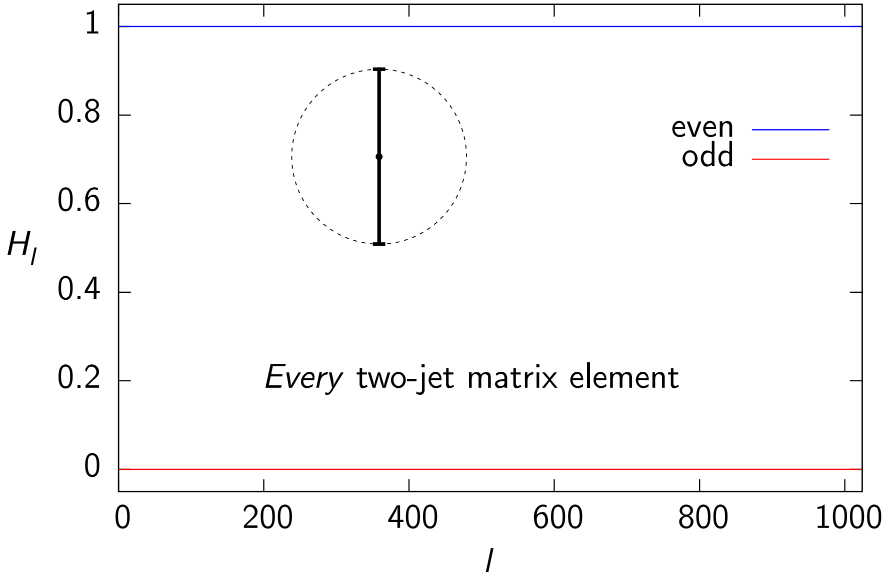
$$\rho(\theta, \phi) = E_{\mathrm{tot}} \sum_{i=1}^{N} f_i \, \delta^2(\theta_i, \phi_i) \quad \left(\text{energy fraction } f_i \equiv \frac{|\vec{p}_i|}{E_{\mathrm{tot}}}\right) .$$

This gives the power spectrum introduced by Fox and Wolfram in 1978

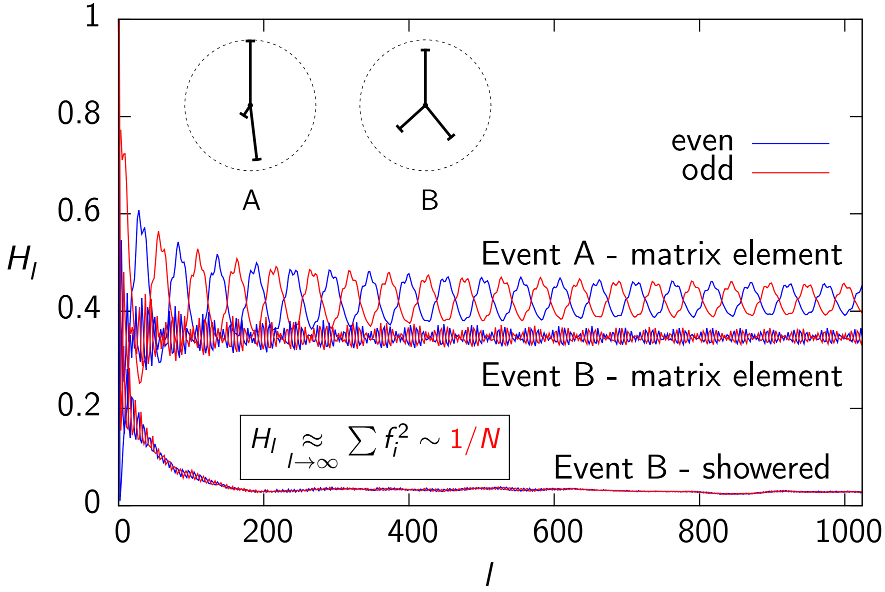
$$H_l = \sum_{i,j} \frac{|\vec{p}_i| \, |\vec{p}_j|}{E_{\mathrm{vis}}^2} P_l(\cos\theta_{ij}) = \sum_{i,j} f_i \, f_j \; P_l(\cos\theta_{ij}) \, .$$

Needs high particle multiplicity: $\underbrace{\text{13 TeV}}_{2017} \gg \underbrace{\text{19 GeV}}_{1978}$ . . . time to revisit H_l.

H_l of simple matrix elements (in the CM frame)



H_l of simple matrix elements (in the CM frame)



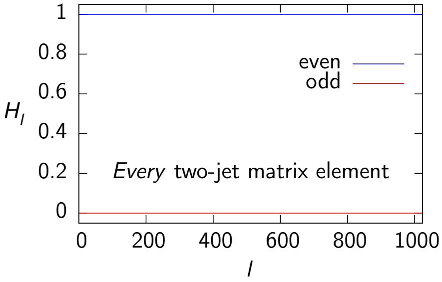
Multiplicity attenuates H_l to a white noise plateau

$H_l \underset{l\to\infty}{\approx} \sum f_i^2 \sim 1/N \quad \Longrightarrow \quad$ The power spectrum flattens to a plateau

---

$\rho(\theta,\phi) = \sum_i f_i \, \delta^3(\hat{p}_i - \hat{r}) \quad \Longrightarrow \quad$ We get H_l from a *discrete* sample .

$\int_{-\infty}^{\infty} \delta(x) e^{-i\,k\,x} \mathrm{d}x = 1 \quad \Longrightarrow \quad$ $\delta(x)$ has uniform power (white noise).



This explains why the 2-jet power spectrum has every even power and no odd power

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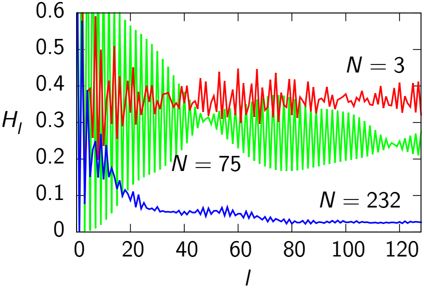
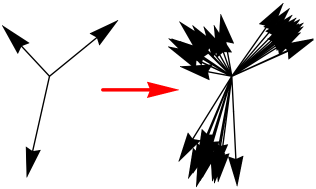
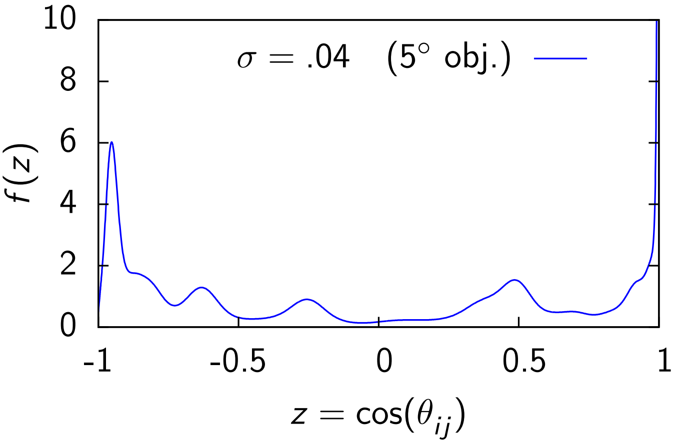


To suppress high-frequency white noise, make particles **extensive**

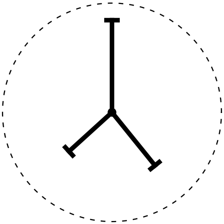


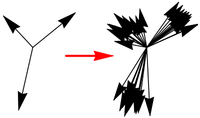
Showered, smeared power spectra

$$H_l = \sum_{i,j} f_i f_j P_l(\cos\theta_{ij}) \qquad f(z) = \sum_l (2l+1) H_l P_l(z)$$



$A_{\text{peak}} \sim f_i f_j$





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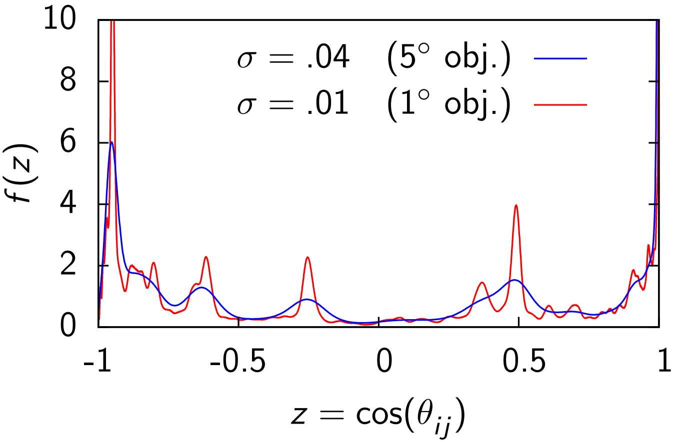

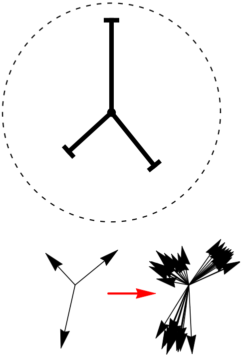

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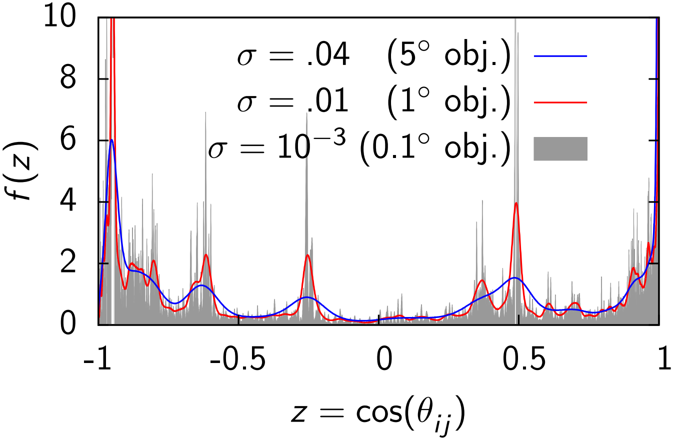

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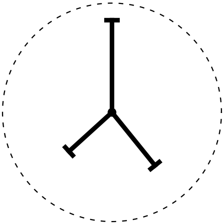

H_l at small l (coarse event shape) is IRC safe.

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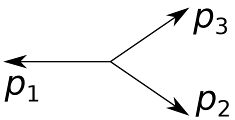
The H_l jet definition

Calculate H_l for *n*-**jet toy system**, fit to H_l for N detected particles.

$$H_l = \sum_{i,j} f_i f_j P_l(\cos\theta_{ij}), \qquad f_i \equiv \frac{E_i}{\sqrt{s}}$$

$$\chi_l = H_l^{\text{reco}} - H_l^{\text{obs}}$$

$$e^+e^- \to 3j, \ \sqrt{s} = 400\ \text{GeV}$$

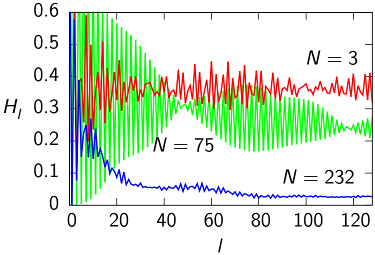


$$\mathbf{p}_j^2 = 0$$

$$\sum \vec{p} = 0$$

Jets without an R parameter!

$$(\boldsymbol{p}_1 + \boldsymbol{p}_2)^2 = s\left((f_1 + f_2)^2 - f_3^2\right)$$

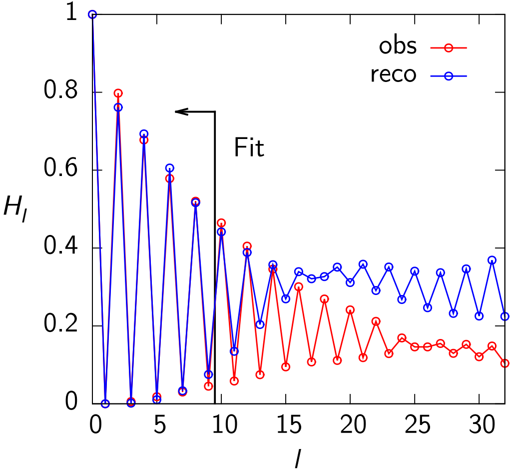


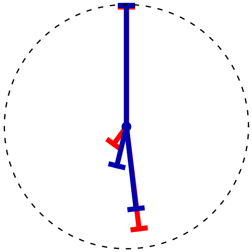
3 jets won't match plateau ($H_l \sim 1/N$ as $l \to \infty$).

- Utility requires $n \ll N$.
- The plateau pressures jet equilibration ($f_i = \frac{1}{n}$).
- Fit to $l_{\max}$, determined by $\chi^2 < \epsilon \max(H_l)$

Fitting a 2-jet-like event

3-jet fit overestimates H_l for $l > \mathcal{O}(10)$.





	ME	Fit showered
f_1	0.49	0.49 ($\pm$0%)
f_2	0.42	0.34 ($-$20%)
f_3	0.09	0.17 ($+$90%)

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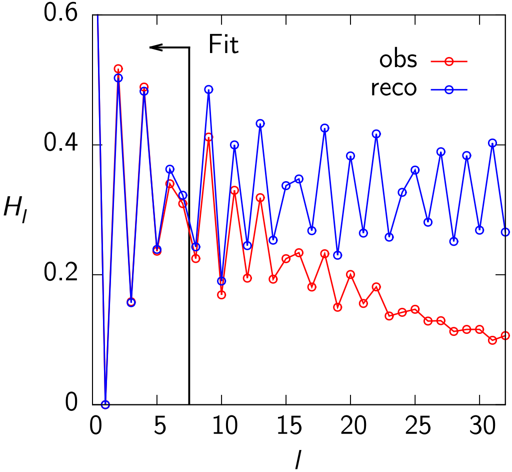
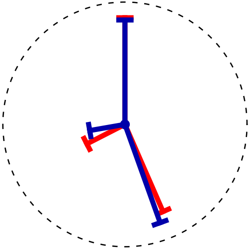
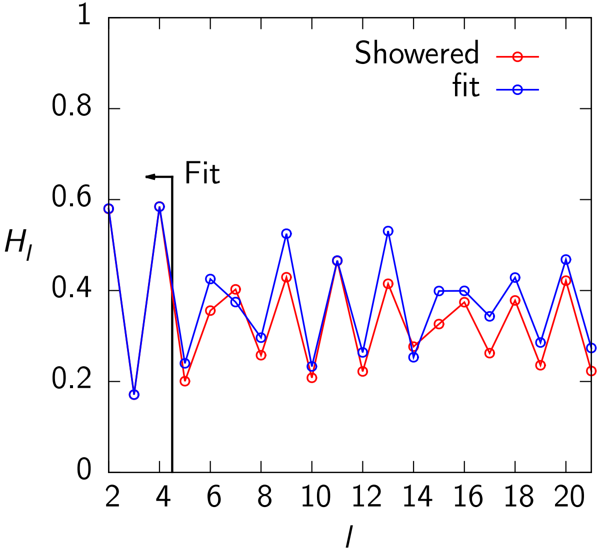
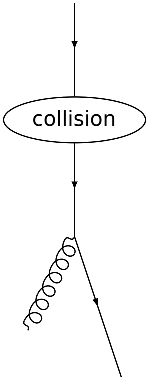




	ME	Fit showered
f_1	0.44	0.45 (+2%)
f_2	0.39	0.40 (+2.5%)
f_3	0.17	0.15 (−11%)

Overfitting (3 d.o.f. per each new jet)





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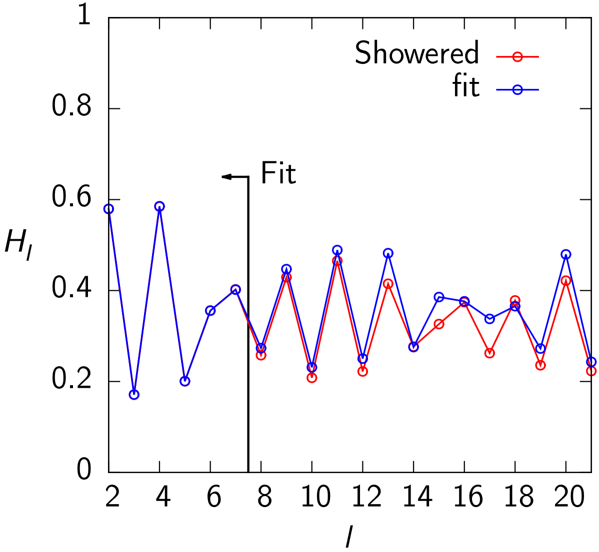

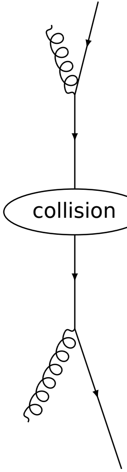

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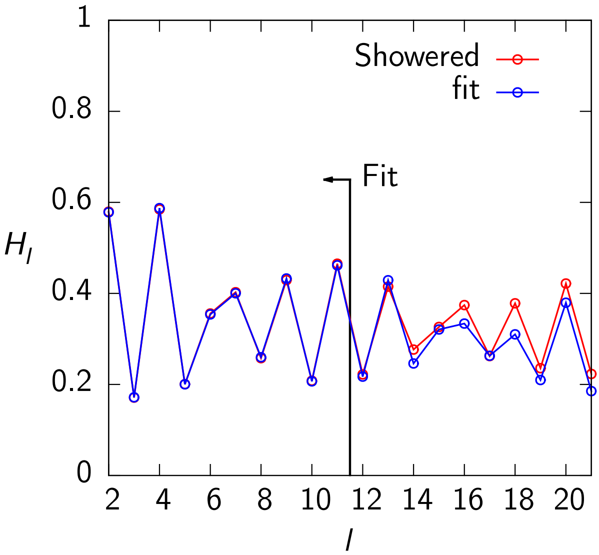

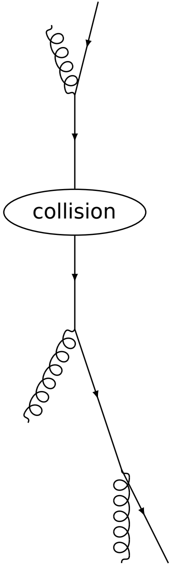

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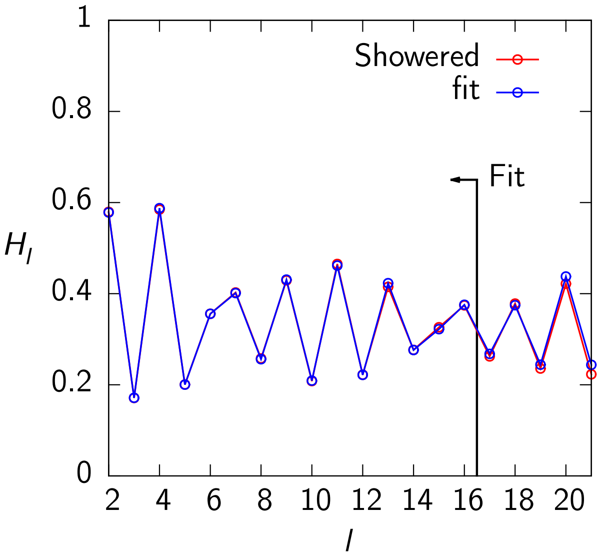
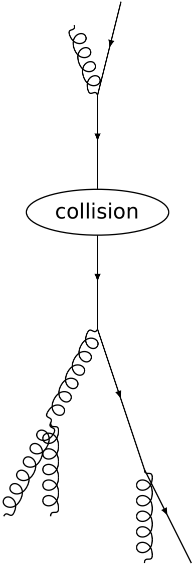
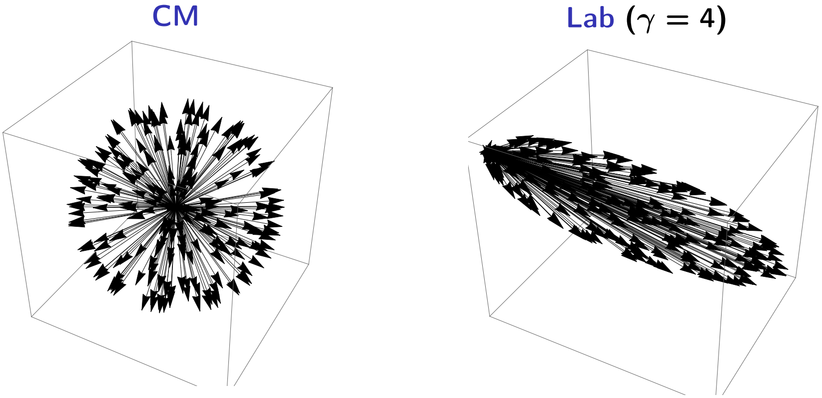




Jet shape

- An n-jet model cannot fit H_l for busy final states ($N \gg n$).
- Sending $n \to N$ fixes the problem, but is highly overfit.
- δ-functions are too thin; jets are extensive!
- **Simplest jet shape**: scalar decay boosted into the lab frame.



Calculating H_l^{reco} for arbitrary shapes

Given some arbitrary density

$$\rho(\hat{r}) = \rho_{(1)}(\hat{r}) + \rho_{(2)}(\hat{r}) + \ldots + \rho_{(n)}(\hat{r}),$$

we can calculate the spectral power *two ways* ($A_l = \frac{1}{E_{\text{tot}}^2} \frac{4\pi}{2l+1}$)

$$H_l = A_l \sum_{m=-l}^{l} |\rho_l^m|^2 = A_l \int \mathrm{d}\Omega \int \mathrm{d}\Omega' P_l(\hat{r} \cdot \hat{r}') \rho(\hat{r}) \rho(\hat{r}')$$

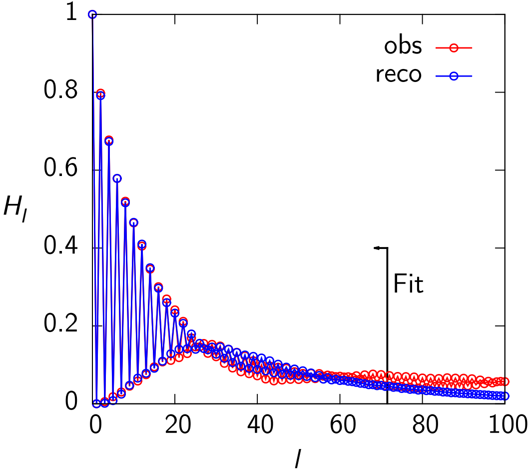
$$= A_l \sum_{m=-l}^{+l} \left({\rho_{(1)}}_l^m {\rho_{(1)}}_l^{m*} + 2{\rho_{(1)}}_l^m {\rho_{(2)}}_l^{m*} + \ldots + {\rho_{(n)}}_l^m {\rho_{(n)}}_l^{m*} \right)$$

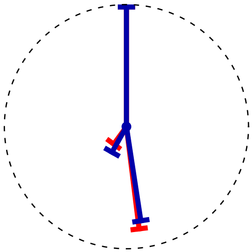
Rotate each pair so $\rho_{(i)} \parallel \hat{z}$. Azimuthal symmetry $\Rightarrow {\rho_{(i)}}_l^m = 0$ for $m \neq 0$.

We only need to compute $\breve{\rho}_{(i)_l} = \int_{-1}^{1} P_l(z)\, \rho_{(i)}(z)\, \mathrm{d}z$

Fitting a 2-jet-like event with massive jets

Jet shapes attenuate H_l





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f_1	0.49	0.49 ($\pm 0\%$)
f_2	0.42	0.39 (-7%)
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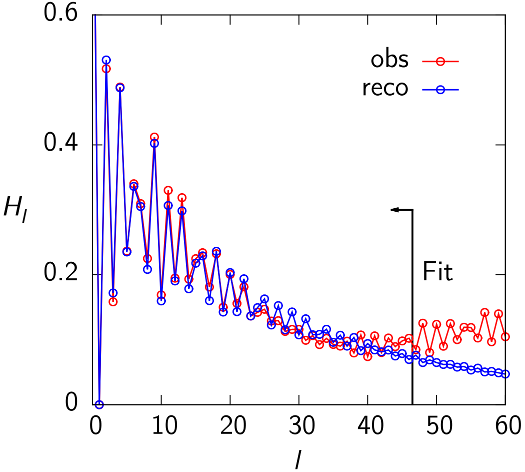
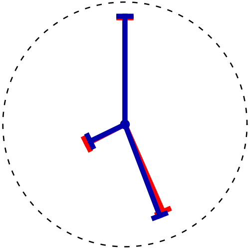




	ME	Fit showered
f_1	0.44	0.44 ($\pm$0%)
f_2	0.39	0.40 (+2.5%)
f_3	0.17	0.16 ($-$6%)

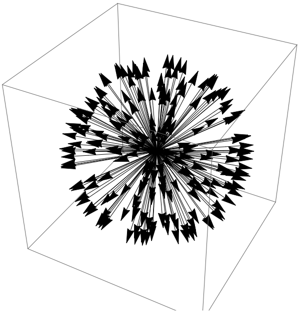
How can we account for pileup in H_l jets?

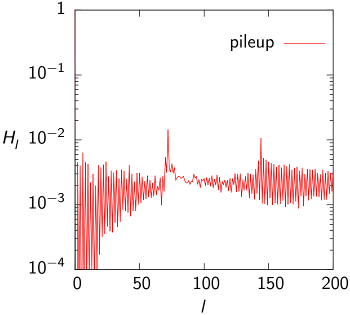
Pileup adds to the energy density ... with a measureable, consistent shape

$$\rho(\hat{r}) = \rho_{\text{hard}}(\hat{r}) + \rho_{\text{pileup}}(\hat{r})$$

- ρ_{pileup} — measured from pileup's H_l (**min-bias** events).
- f_{PU} — pileup energy fraction. Free parameter during fit.

We choose a simple pileup model (isotropic) for e^+e^- simulations.

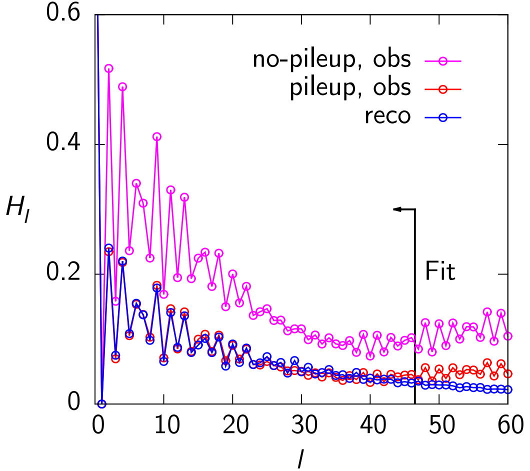


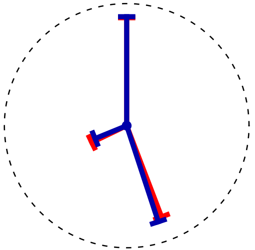


3-jet-like event with extreme pileup ($f_{\text{PU}} = 1/3$)

Jets still visible through extreme pileup:

$$(H_l^{\text{obs}})_{\text{pileup}} = (1 - f_{\text{PU}})^2 H_l^{\text{obs}}$$





	no PU	Fit w/ PU
$\tilde{f}_1$	0.44	0.45 (+2.3%)
$\tilde{f}_2$	0.40	0.41 (+2.5%)
$\tilde{f}_3$	0.16	0.14 (−12%)
f_{PU}	10^{-2}	0.31

normalized $\tilde{f} = f/(1 - f_{PU})$

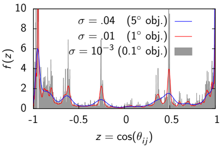
Conclusion

The high multiplicity at modern colliders gives H_l the ability to extract final-state correlations. Our initial study uses the H_l jet definition to extract the **jet-like structure (2-jet, 3-jet)**:

- Started with massless, δ-function jet model — good fit.
- Model improved by adding jet mass — *better* fit.
  - Higher moments accessible
  - We can ask questions about jet shape
- Pileup contribution can be fit to isolate signal
  - The shape of the pileup energy density ρ can be **measured** in min-bias.
  - H_l harnesses pileup correlations to make in situ measurement

What's next?

- Apply H_l to a proton collider physics.
- Expand H_l jet fit to identify jet substructure
  - Reconstruct top layer of showering.
  - Substructure is already visible at low l!



THANK YOU!